Q1.

The diagram shows a metal plate ABCDEF which has been made by removing the two shaded regions from a circle of radius 10 cm and centre O. The parallel edges AB and ED are both of length 12 cm.

(i) Show that angle DOE is 1.287 radians, correct to 4 significant figures. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

Q2.

In the diagram, OAB is an isosceles triangle with OA = OB and angle AOB = 2θ radians. Arc PST has centre O and radius r, and the line ASB is a tangent to the arc PST at S.

(i) Find the total area of the shaded regions in terms of r and θ. [4]

(ii) In the case where θ = \(\frac{1}{3}\pi\) and r = 6, find the total perimeter of the shaded regions, leaving your answer in terms of \(\sqrt{3}\) and \(\pi\). [5]

Q3.
Q4.

In the diagram, $AB$ is an arc of a circle, centre $O$ and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line $AX$ is a tangent to the circle at $A$, and $OBX$ is a straight line.

(i) Show that the exact length of $AX$ is $6\sqrt{3}$ cm. \[1\]

Find, in terms of $\pi$ and $\sqrt{3}$,

(ii) the area of the shaded region, \[3\]

(iii) the perimeter of the shaded region. \[4\]

Q5.

In the diagram, $ABC$ is an equilateral triangle of side 2 cm. The mid-point of $BC$ is $Q$. An arc of a circle with centre $A$ touches $BC$ at $Q$, and meets $AB$ at $P$ and $AC$ at $R$. Find the total area of the shaded regions, giving your answer in terms of $\pi$ and $\sqrt{3}$. \[5\]
In the diagram, $AB$ is an arc of a circle with centre $O$ and radius $r$. The line $XB$ is a tangent to the circle at $B$ and $A$ is the mid-point of $OX$.

(i) Show that angle $AOB = \frac{1}{3}\pi$ radians. \[2\]

Express each of the following in terms of $r$, $\pi$ and $\sqrt{3}$:

(ii) the perimeter of the shaded region. \[3\]

(iii) the area of the shaded region. \[2\]

Q6.

In the diagram, $OAB$ is a sector of a circle with centre $O$ and radius 8 cm. Angle $BOA$ is $\alpha$ radians. $OAC$ is a semicircle with diameter $OA$. The area of the semicircle $OAC$ is twice the area of the sector $OAB$.

(i) Find $\alpha$ in terms of $\pi$. \[3\]

(ii) Find the perimeter of the complete figure in terms of $\pi$. \[2\]

Q7.
Q8. The diagram shows a circle \( C \) with centre \( O \) and radius 3 cm. The radii \( OP \) and \( OQ \) are extended to \( S \) and \( R \) respectively so that \( OQS \) is a sector of a circle with centre \( O \). Given that \( PE = 6 \) cm and that the area of the shaded region is equal to the area of circle \( C \).

(i) show that angle \( POQ = \frac{1}{3} \pi \) radians. \( \text{[3]} \)

(ii) find the perimeter of the shaded region. \( \text{[2]} \)

Q9. The diagram shows a semicircle \( ABC \) with centre \( O \) and radius 6 cm. The point \( B \) is such that angle \( BOA \) is 90° and \( BD \) is an arc of a circle with centre \( A \). Find

(i) the length of the arc \( BD \). \( \text{[4]} \)

(ii) the area of the shaded region. \( \text{[3]} \)
Q10.

The diagram shows two circles, $C_1$ and $C_2$, touching at the point $T$. Circle $C_1$ has centre $P$ and radius 8 cm; circle $C_2$ has centre $Q$ and radius 2 cm. Points $R$ and $S$ lie on $C_1$ and $C_2$ respectively, and $RS$ is a tangent to both circles.

(i) Show that $RS = 8$ cm. [2]

(ii) Find angle $RPQ$ in radians correct to 4 significant figures. [2]

(iii) Find the area of the shaded region. [4]

Q11.

The diagram shows a rhombus $ABCD$. Points $P$ and $Q$ lie on the diagonal $AC$ such that $BPD$ is an arc of a circle with centre $C$ and $BQD$ is an arc of a circle with centre $A$. Each side of the rhombus has length 5 cm and angle $BAD = 1.2$ radians.

(i) Find the area of the shaded region $BPDQ$. [4]

(ii) Find the length of $PQ$. [4]
The diagram represents a metal plate $OABC$, consisting of a sector $OAB$ of a circle with centre $O$ and radius $r$, together with a triangle $OCB$ which is right-angled at $C$. Angle $AOB = \theta$ radians and $OC$ is perpendicular to $OA$.

(i) Find an expression in terms of $r$ and $\theta$ for the perimeter of the plate. \hfill [3]

(ii) For the case where $r = 10$ and $\theta = \frac{1}{2}\pi$, find the area of the plate. \hfill [3]

Q12.

4

In the diagram, $ABCD$ is a parallelogram with $AB = BD = DC = 10$ cm and angle $ABD = 0.8$ radians. $APD$ and $BQC$ are arcs of circles with centres $B$ and $D$ respectively.

(i) Find the area of the parallelogram $ABCD$. \hfill [2]

(ii) Find the area of the complete figure $ABQCDP$. \hfill [2]

(iii) Find the perimeter of the complete figure $ABQCDP$. \hfill [2]

Q13.
Q14.

The diagram shows a sector $OAB$ of a circle with centre $O$ and radius $r$. Angle $AOB$ is $\theta$ radians. The point $C$ on $OA$ is such that $BC$ is perpendicular to $OA$. The point $D$ is on $BC$ and the circular arc $AD$ has centre $C$.

(i) Find $AC$ in terms of $r$ and $\theta$. \[ \text{[1]} \]

(ii) Find the perimeter of the shaded region $ABD$ when $\theta = \frac{1}{2}\pi$ and $r = 4$, giving your answer as an exact value. \[ \text{[6]} \]

Q15.

In the diagram, $D$ lies on the side $AB$ of triangle $ABC$ and $CD$ is an arc of a circle with centre $A$ and radius 2 cm. The line $BC$ is of length $2\sqrt{3}$ cm and is perpendicular to $AC$. Find the area of the shaded region $BDC$, giving your answer in terms of $\pi$ and $\sqrt{3}$. \[ \text{[4]} \]
The diagram shows a metal plate made by fixing together two pieces, \( \triangle ABCD \) (shaded) and \( \triangle AED \) (unshaded). The piece \( \triangle ABCD \) is a minor sector of a circle with centre \( O \) and radius \( 2r \). The piece \( \triangle AED \) is a major sector of a circle with centre \( O \) and radius \( r \). Angle \( AOD \) is \( \alpha \) radians. Simplifying your answers where possible, find, in terms of \( \alpha \), \( \pi \) and \( r \),

(i) the perimeter of the metal plate, \[3\]

(ii) the area of the metal plate. \[3\]

It is now given that the shaded and unshaded pieces are equal in area.

(iii) Find \( \alpha \) in terms of \( \pi \). \[2\]

Q16.

The diagram shows sector \( \triangle AOB \) with centre \( O \) and radius 11 cm. Angle \( AOB = \alpha \) radians. Points \( C \) and \( D \) lie on \( OA \) and \( OB \) respectively. Arc \( CD \) has centre \( O \) and radius 5 cm.

(i) The area of the shaded region \( ABDC \) is equal to \( k \) times the area of the unshaded region \( OCD \). Find \( k \). \[3\]

(ii) The perimeter of the shaded region \( ABDC \) is equal to twice the perimeter of the unshaded region \( OCD \). Find the exact value of \( \alpha \). \[4\]