Q1.

1

The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]

Q2.

5

The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{3x-2}}$. Given that the curve passes through the point $P (2, 11)$, find

(i) the equation of the normal to the curve at $P$.            [3]
(ii) the equation of the curve.                              [4]

Q3.

9

The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at $M$. The line $y = 5$ intersects the curve at the points $A$ and $B$.

(i) Find the coordinates of $A, B$ and $M$.               [5]

(ii) Find the volume obtained when the shaded region is rotated through $360^\circ$ about the $x$-axis. [6]

Q4.

3

(i) Sketch the curve $y = (x-2)^2$.                        [1]

(ii) The region enclosed by the curve, the $x$-axis and the $y$-axis is rotated through $360^\circ$ about the $x$-axis. Find the volume obtained, giving your answer in terms of $\pi$. [4]
Q5.

7 A curve is such that \( \frac{dy}{dx} = \frac{3}{(1+2x)^2} \) and the point \( (1, \frac{1}{2}) \) lies on the curve.

(i) Find the equation of the curve. \[4\]

(ii) Find the set of values of \( x \) for which the gradient of the curve is less than \( \frac{1}{3} \). \[3\]

Q6.

9 A curve is such that \( \frac{dy}{dx} = \frac{1}{\sqrt{x}} - 1 \) and \( P (9, 5) \) is a point on the curve.

(i) Find the equation of the curve. \[4\]

(ii) Find the coordinates of the stationary point on the curve. \[3\]

(iii) Find an expression for \( \frac{d^2y}{dx^2} \) and determine the nature of the stationary point. \[2\]

(iv) The normal to the curve at \( P \) makes an angle of \( \tan^{-1} k \) with the positive \( x \)-axis. Find the value of \( k \). \[2\]

Q7.

11

The diagram shows the line \( y = 1 \) and part of the curve \( y = \frac{2}{\sqrt{x+1}} \).

(i) Show that the equation \( y = \frac{2}{\sqrt{x+1}} \) can be written in the form \( x = \frac{4}{y^2} - 1 \). \[1\]

(ii) Find \( \int \left( \frac{4}{y^2} - 1 \right) \, dy \). Hence find the area of the shaded region. \[5\]

(iii) The shaded region is rotated through 360\(^\circ\) about the \( y \)-axis. Find the exact value of the volume of revolution obtained. \[5\]

Q8.
The diagram shows part of the curve \( x = \frac{8}{y^2} - 2 \), crossing the y-axis at the point \( A \). The point \( B(6, 1) \) lies on the curve. The shaded region is bounded by the curve, the y-axis, and the line \( y = 1 \). Find the exact volume obtained when this shaded region is rotated through \( 360^\circ \) about the y-axis. \[ 6 \]

**Q9.**

9 A curve is such that \( \frac{dy}{dx^2} = -4x \). The curve has a maximum point at \( (2, 1) \).

(i) Find the equation of the curve. \[ 6 \]

A point \( P \) moves along the curve in such a way that the \( x \)-coordinate is increasing at 0.05 units per second.

(ii) Find the rate at which the \( y \)-coordinate is changing when \( x = 3 \), stating whether the \( y \)-coordinate is increasing or decreasing. \[ 2 \]

**Q10.**
The diagram shows part of the curve \( y = (x - 2)^4 \) and the point \( A (1, 1) \) on the curve. The tangent at \( A \) cuts the \( x \)-axis at \( B \) and the normal at \( A \) cuts the \( y \)-axis at \( C \).

(i) Find the coordinates of \( B \) and \( C \). [6]

(ii) Find the distance \( AC \), giving your answer in the form \( \frac{\sqrt{a}}{b} \), where \( a \) and \( b \) are integers. [2]

(iii) Find the area of the shaded region. [4]

Q11.

A curve is such that \( \frac{dy}{dx} = \sqrt{2x + 5} \) and \( (2, 5) \) is a point on the curve. Find the equation of the curve. [4]

Q12.
The diagram shows part of the curve \( y = \frac{8}{\sqrt{x}} - x \) and points \( A(1, 7) \) and \( B(4, 0) \) which lie on the curve. The tangent to the curve at \( B \) intersects the line \( x = 1 \) at the point \( C \).

(i) Find the coordinates of \( C \). [4]

(ii) Find the area of the shaded region. [5]

Q13.

A curve is such that \( \frac{dy}{dx} = k - 2x \), where \( k \) is a constant.

(i) Given that the tangents to the curve at the points where \( x = 2 \) and \( x = 3 \) are perpendicular, find the value of \( k \). [4]

(ii) Given also that the curve passes through the point \((4, 9)\), find the equation of the curve. [3]

Q14.

Find \( \int \left(x + \frac{1}{x}\right)^3 \, dx \). [3]

Q15.

The equation of a curve is \( y = \frac{2 - x}{2 - x} \).

(i) Find an expression for \( \frac{dy}{dx} \) and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line \( x = 1 \) is rotated through \( 360^\circ \) about the \( x \)-axis. [4]

(iii) Find the set of values of \( k \) for which the line \( y = x + k \) intersects the curve at two distinct points. [4]

Q16.
The diagram shows parts of the curves \( y = 9 - x^3 \) and \( y = \frac{8}{x^3} \) and their points of intersection \( P \) and \( Q \). The \( x \)-coordinates of \( P \) and \( Q \) are \( a \) and \( b \) respectively.

(i) Show that \( x = a \) and \( x = b \) are roots of the equation \( x^6 - 9x^3 + 8 = 0 \). Solve this equation and hence state the value of \( a \) and the value of \( b \). \[4\]

(ii) Find the area of the shaded region between the two curves. \[5\]

(iii) The tangents to the two curves at \( x = c \) (where \( a < c < b \)) are parallel to each other. Find the value of \( c \). \[4\]
Q18.

The diagram shows the curve \( y = \sqrt{1 + 2x} \) meeting the \( x \)-axis at \( A \) and the \( y \)-axis at \( B \). The \( y \)-coordinate of the point \( C \) on the curve is 3.

(i) Find the coordinates of \( B \) and \( C \). [2]

(ii) Find the equation of the normal to the curve at \( C \). [4]

(iii) Find the volume obtained when the shaded region is rotated through 360° about the \( y \)-axis. [5]

Q19.

The diagram shows the line \( y = x + 1 \) and the curve \( y = \sqrt{x + 1} \), meeting at \((-1, 0)\) and \((0, 1)\).

(i) Find the area of the shaded region. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the \( y \)-axis. [7]
Q20.

A curve is such that \( \frac{dy}{dx} = -\frac{8}{x^3} - 1 \) and the point (2, 4) lies on the curve. Find the equation of the curve. \([4]\)

Q21.

The diagram shows the curve \( y^2 = 2x - 1 \) and the straight line \( 3y = 2x - 1 \). The curve and straight line intersect at \( x = \frac{1}{2} \) and \( x = \alpha \), where \( \alpha \) is a constant.

\( \text{(i)} \) Show that \( \alpha = 5 \). \([2]\)

\( \text{(ii)} \) Find, showing all necessary working, the area of the shaded region. \([6]\)
The diagram shows the curve with equation \( y = x(x - 2)^2 \). The minimum point on the curve has coordinates \((a, 0)\) and the \(x\)-coordinate of the maximum point is \(b\), where \(a\) and \(b\) are constants.

(i) State the value of \(a\). \(\boxed{[1]}\)

(ii) Find the value of \(b\). \(\boxed{[4]}\)

(iii) Find the area of the shaded region. \(\boxed{[4]}\)

(iv) The gradient, \(\frac{dy}{dx}\), of the curve has a minimum value \(m\). Find the value of \(m\). \(\boxed{[4]}\)

Q22.

A curve has equation \( y = f(x) \). It is given that \( f'(x) = \frac{1}{\sqrt{x + 6}} + \frac{6}{x^2} \) and that \( f(3) = 1 \). Find \( f(x) \). \(\boxed{[5]}\)

Q23.

The diagram shows the curve \( y = (3 - 2x)^3 \) and the tangent to the curve at the point \((\frac{1}{2}, 8)\).

(i) Find the equation of this tangent, giving your answer in the form \( y = nx + c \). \(\boxed{[5]}\)

(ii) Find the area of the shaded region. \(\boxed{[6]}\)
Q24.

A curve has equation \( y = f(x) \). It is given that \( f'(x) = x^{-\frac{3}{2}} + 1 \) and that \( f(4) = 5 \). Find \( f(x) \). \[4\]

Q25.

The diagram shows the curve \( y = \sqrt{x^4 + 4x + 4} \).

(i) Find the equation of the tangent to the curve at the point (0, 2). \[4\]

(ii) Show that the \( x \)-coordinates of the points of intersection of the line \( y = x + 2 \) and the curve are given by the equation \( (x + 2)^2 = x^4 + 4x + 4 \). Hence find these \( x \)-coordinates. \[4\]

(iii) The region shaded in the diagram is rotated through \( 360^\circ \) about the \( x \)-axis. Find the volume of revolution. \[4\]