Q1.

10

The diagram shows the parallelogram $OABC$. Given that $\overrightarrow{OA} = i + 3j + 3k$ and $\overrightarrow{OC} = 3i - j + k$, find

(i) the unit vector in the direction of $\overrightarrow{OB}$, [3]
(ii) the acute angle between the diagonals of the parallelogram, [5]
(iii) the perimeter of the parallelogram, correct to 1 decimal place. [3]

Q2.

6 Relative to an origin $O$, the position vectors of the points $A$, $B$ and $C$ are given by

$\overrightarrow{OA} = i - 2j + 4k$, $\overrightarrow{OB} = 3i + 2j + 8k$, $\overrightarrow{OC} = -i - 2j + 10k$.

(i) Use a scalar product to find angle $ABC$. [6]
(ii) Find the perimeter of triangle $ABC$, giving your answer correct to 2 decimal places. [2]

Q3.
Q4.

The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length $6$ cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges $AB$ and $DC$ are of lengths $5$ cm and $2$ cm respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $AD$, $AP$ and $AB$ respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. 

(ii) Use a scalar product to calculate angle $PCQ$.

Q5.

In the diagram, $OABCDEF$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $OA$, $OC$ and $OD$ respectively. The point $P$ is the mid-point of $DG$, $Q$ is the centre of the square face $CBFG$ and $R$ lies on $AB$ such that $AR = 4$ cm.

(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.

(ii) Use a scalar product to find angle $RQP$. 

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6 Two vectors \( \mathbf{u} \) and \( \mathbf{v} \) are such that
\[
\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 2 \\ p - 1 \\ 2p + 1 \end{pmatrix},
\]
where \( p \) is a constant.

(i) Find the values of \( p \) for which \( \mathbf{u} \) is perpendicular to \( \mathbf{v} \). [3]

(ii) For the case where \( p = 1 \), find the angle between the directions of \( \mathbf{u} \) and \( \mathbf{v} \). [4]

Q6.

2 Relative to an origin \( O \), the position vectors of the points \( A, B \) and \( C \) are given by
\[
\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.
\]

Find

(i) the unit vector in the direction of \( \overrightarrow{AB} \). [3]

(ii) the value of the constant \( p \) for which angle \( BOC = 90^\circ \). [2]

Q7.

6 Relative to an origin \( O \), the position vectors of three points \( A, B \) and \( C \), are given by
\[
\overrightarrow{OA} = i + 2p j + qk, \quad \overrightarrow{OB} = qj - 2p k \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)i + 2pj + qk,
\]
where \( p \) and \( q \) are constants.

(i) Show that \( \overrightarrow{OA} \) is perpendicular to \( \overrightarrow{OC} \) for all non-zero values of \( p \) and \( q \). [2]

(ii) Find the magnitude of \( \overrightarrow{OA} \) in terms of \( p \) and \( q \). [2]

(iii) For the case where \( p = 3 \) and \( q = 2 \), find the unit vector parallel to \( \overrightarrow{OA} \). [3]

Q8.
The diagram shows a parallelogram \( OABC \) in which
\[
\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.
\]

(i) Use a scalar product to find angle \( BOC \). [6]

(ii) Find a vector which has magnitude 35 and is parallel to the vector \( \overrightarrow{OC} \). [2]

Q9.

Relative to an origin \( O \), the position vectors of the points \( A, B \) and \( C \) are given by
\[
\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}.
\]

(i) Find angle \( AOB \). [4]

(ii) Find the vector which is in the same direction as \( \overrightarrow{AC} \) and has magnitude 30. [3]

(iii) Find the value of the constant \( p \) for which \( \overrightarrow{OA} + p \overrightarrow{OB} \) is perpendicular to \( \overrightarrow{OC} \). [3]

Q10.
The diagram shows a pyramid $OABC$ with a horizontal base $OAB$ where $OA = 6\text{ cm}$, $OB = 8\text{ cm}$ and angle $AOB = 90^\circ$. The point $C$ is vertically above $O$ and $OC = 10\text{ cm}$. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $OA$, $OB$ and $OC$ as shown.

Use a scalar product to find angle $ACB$. [6]

Q11.

10

The diagram shows triangle $OAB$, in which the position vectors of $A$ and $B$ with respect to $O$ are given by

$$\overrightarrow{OA} = 2\mathbf{i} + j - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = -3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}.$$

$C$ is a point on $OA$ such that $\overrightarrow{OC} = p\overrightarrow{OA}$, where $p$ is a constant.

(i) Find angle $AOB$. [4]

(ii) Find $\overrightarrow{BC}$ in terms of $p$ and vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

(iii) Find the value of $p$ given that $BC$ is perpendicular to $OA$. [4]

Q12.
Q13.

Relative to an origin $O$, the position vectors of points $A$ and $B$ are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

(i) Use a scalar product to find angle $\angle BOA$. [4]

The point $C$ is the mid-point of $AB$. The point $D$ is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

(ii) Find $\overrightarrow{DC}$. [4]

Q14.

The position vectors of points $A$ and $B$ relative to an origin $O$ are $\mathbf{a}$ and $\mathbf{b}$ respectively. The position vectors of points $C$ and $D$ relative to $O$ are $3\mathbf{a}$ and $2\mathbf{b}$ respectively. It is given that

\[
\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}.
\]

(i) Find the unit vector in the direction of $\overrightarrow{CD}$. [3]

(ii) The point $E$ is the mid-point of $CD$. Find angle $\angle EOD$. [6]

Q15.

The position vectors of points $A$ and $B$ relative to an origin $O$ are given by

\[
\overrightarrow{OA} = \begin{pmatrix} p \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix},
\]

where $p$ is a constant.

(i) In the case where $OAB$ is a straight line, state the value of $p$ and find the unit vector in the direction of $\overrightarrow{OA}$. [3]

(ii) In the case where $OA$ is perpendicular to $AB$, find the possible values of $p$. [5]

(iii) In the case where $p = 3$, the point $C$ is such that $OABC$ is a parallelogram. Find the position vector of $C$. [2]
Q16.

The diagram shows a pyramid $OABCD$ in which the vertical edge $OD$ is 3 units in length. The point $E$ is the centre of the horizontal rectangular base $OABC$. The sides $OA$ and $AB$ have lengths of 6 units and 4 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively.

(i) Express each of the vectors $\overrightarrow{DE}$ and $\overrightarrow{EB}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. \hspace{1cm} [2]

(ii) Use a scalar product to find angle $\angle BDE$. \hspace{1cm} [4]

Q17.

The diagram shows a pyramid $OABC$ in which the edge $OC$ is vertical. The horizontal base $OAB$ is a triangle, right-angled at $O$, and $D$ is the mid-point of $AB$. The edges $OA$, $OE$ and $OC$ have lengths of 8 units, 6 units and 10 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively.

(i) Express each of the vectors $\overrightarrow{OD}$ and $\overrightarrow{BC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. \hspace{1cm} [2]

(ii) Use a scalar product to find angle $\angle OCD$. \hspace{1cm} [4]