**Chapter 7 Vectors**

May/June 2002

8  The straight line $l$ passes through the points $A$ and $B$ whose position vectors are $i + k$ and $4i - j + 3k$ respectively. The plane $p$ has equation $x + 3y - 2z = 3$.

(i) Given that $l$ intersects $p$, find the position vector of the point of intersection.  

(ii) Find the equation of the plane which contains $l$ and is perpendicular to $p$, giving your answer in the form $ax + by + cz = 1$.  

Oct/Nov 2002

10  With respect to the origin $O$, the points $A$, $B$, $C$, $D$ have position vectors given by

$$
\overrightarrow{OA} = 4i + k, \quad \overrightarrow{OB} = 5i - 2j - 2k, \quad \overrightarrow{OC} = i + j, \quad \overrightarrow{OD} = -i - 4k.
$$

(i) Calculate the acute angle between the lines $AB$ and $CD$.  

(ii) Prove that the lines $AB$ and $CD$ intersect.  

(iii) The point $P$ has position vector $i + 5j + 6k$. Show that the perpendicular distance from $P$ to the line $AB$ is equal to $\sqrt{3}$.  

May/June 2003

9  Two planes have equations $x + 2y - 2z = 2$ and $2x - 3y + 6z = 3$. The planes intersect in the straight line $l$.

(i) Calculate the acute angle between the two planes.  

(ii) Find a vector equation for the line $l$.  

Oct/Nov 2003

10  The lines $l$ and $m$ have vector equations

$$
r = i - 2k + s(2i + j + 3k) \quad \text{and} \quad r = 6i - 5j + 4k + t(i - 2j + k)
$$

respectively.

(i) Show that $l$ and $m$ intersect, and find the position vector of their point of intersection.  

(ii) Find the equation of the plane containing $l$ and $m$, giving your answer in the form $ax + by + cz = d$.  

May/June 2004

11 With respect to the origin $O$, the points $P$, $Q$, $R$, $S$ have position vectors given by

\[
\overrightarrow{OP} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OQ} = -2\mathbf{i} + 4\mathbf{j}, \quad \overrightarrow{OR} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OS} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}.
\]

(i) Find the equation of the plane containing $P$, $Q$ and $R$, giving your answer in the form $ax + by + cz = d$. \[6\]

(ii) The point $N$ is the foot of the perpendicular from $S$ to this plane. Find the position vector of $N$ and show that the length of $SN$ is 7. \[6\]

Oct/Nov 2004

9 The lines $l$ and $m$ have vector equations

\[
\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})
\]

respectively.

(i) Show that $l$ and $m$ do not intersect. \[4\]

The point $P$ lies on $l$ and the point $Q$ has position vector $2\mathbf{i} - \mathbf{k}$.

(ii) Given that the line $PQ$ is perpendicular to $l$, find the position vector of $P$. \[4\]

(iii) Verify that $Q$ lies on $m$ and that $PQ$ is perpendicular to $m$. \[2\]

May/June 2005

10 With respect to the origin $O$, the points $A$ and $B$ have position vectors given by

\[
\overrightarrow{OA} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = \mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.
\]

The line $l$ has vector equation $\mathbf{r} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k} + s(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Prove that the line $l$ does not intersect the line through $A$ and $B$. \[5\]

(ii) Find the equation of the plane containing $l$ and the point $A$, giving your answer in the form $ax + by + cz = d$. \[6\]

Oct/Nov 2005

10 The straight line $l$ passes through the points $A$ and $B$ with position vectors

\[
2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}
\]

respectively. This line intersects the plane $p$ with equation $x - 2y + 2z = 6$ at the point $C$.

(i) Find the position vector of $C$. \[4\]

(ii) Find the acute angle between $l$ and $p$. \[4\]

(iii) Show that the perpendicular distance from $A$ to $p$ is equal to 2. \[3\]
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May/June 2006

10  The points \( A \) and \( B \) have position vectors, relative to the origin \( O \), given by

\[
\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.
\]

The line \( l \) passes through \( A \) and is parallel to \( \overrightarrow{OB} \). The point \( N \) is the foot of the perpendicular from \( B \) to \( l \).

(i) State a vector equation for the line \( l \). [1]

(ii) Find the position vector of \( N \) and show that \( BN = 3 \). [6]

(iii) Find the equation of the plane containing \( A, B \) and \( N \), giving your answer in the form \( ax + by + cz = d \). [5]

Oct/Nov 2006

7  The line \( l \) has equation \( \mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \). The plane \( p \) has equation \( x + 2y + 3z = 5 \).

(i) Show that the line \( l \) lies in the plane \( p \). [3]

(ii) A second plane is perpendicular to the plane \( p \), parallel to the line \( l \) and contains the point with position vector \( 2\mathbf{i} + \mathbf{j} + 4\mathbf{k} \). Find the equation of this plane, giving your answer in the form \( ax + by + cz = d \). [6]

May/June 2007

9

The diagram shows a set of rectangular axes \( Ox, Oy \) and \( Oz \), and three points \( A, B \) and \( C \) with position vectors \( \overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \) and \( \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \).

(i) Find the equation of the plane \( ABC \), giving your answer in the form \( ax + by + cz = d \). [6]

(ii) Calculate the acute angle between the planes \( ABC \) and \( OAB \). [4]
Oct/Nov 2007

10 The straight line \( l \) has equation \( \mathbf{r} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) \). The plane \( p \) has equation \( (\mathbf{r} - 3\mathbf{i}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = 0 \). The line \( l \) intersects the plane \( p \) at the point \( A \).

(i) Find the position vector of \( A \). [3]

(ii) Find the acute angle between \( l \) and \( p \). [4]

(iii) Find a vector equation for the line which lies in \( p \), passes through \( A \) and is perpendicular to \( l \). [5]

May/June 2008

10 The points \( A \) and \( B \) have position vectors, relative to the origin \( O \), given by

\[
\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.
\]

The line \( l \) has vector equation

\[
\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.
\]

(i) Show that \( l \) does not intersect the line passing through \( A \) and \( B \). [4]

(ii) The point \( P \) lies on \( l \) and is such that angle \( PAB \) is equal to \( 60^\circ \). Given that the position vector of \( P \) is \((1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}\), show that \( 3t^2 + 7t + 2 = 0 \). Hence find the only possible position vector of \( P \). [6]

Oct/Nov 2008

7 Two planes have equations \( 2x - y - 3z = 7 \) and \( x + 2y + 2z = 0 \).

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for their line of intersection. [6]

May/June 2009

9 The line \( l \) has equation \( \mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \). It is given that \( l \) lies in the plane with equation \( 2x + by + cz = 1 \), where \( b \) and \( c \) are constants.

(i) Find the values of \( b \) and \( c \). [6]

(ii) The point \( P \) has position vector \( 2\mathbf{j} + 4\mathbf{k} \). Show that the perpendicular distance from \( P \) to \( l \) is \( \sqrt{5} \). [5]
6 With respect to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by
\[ \overrightarrow{OA} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}. \]
The mid-point of $AB$ is $M$. The point $N$ lies on $AC$ between $A$ and $C$ and is such that $AN = 2NC$.

(i) Find a vector equation of the line $MN$. \[4\]

(ii) It is given that $MN$ intersects $BC$ at the point $P$. Find the position vector of $P$. \[4\]

Oct/Nov 2009/32

10 The plane $p$ has equation $2x - 3y + 6z = 16$. The plane $q$ is parallel to $p$ and contains the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.

(i) Find the equation of $q$, giving your answer in the form $ax + by + cz = d$. \[2\]

(ii) Calculate the perpendicular distance between $p$ and $q$. \[3\]

(iii) The line $l$ is parallel to the plane $p$ and also parallel to the plane with equation $x - 2y + 2z = 5$. Given that $l$ passes through the origin, find a vector equation for $l$. \[5\]

May/June 2010/31

10 The lines $l$ and $m$ have vector equations
\[ \mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k}) \]
respectively.

(i) Show that $l$ and $m$ intersect. \[4\]

(ii) Calculate the acute angle between the lines. \[3\]

(iii) Find the equation of the plane containing $l$ and $m$, giving your answer in the form $ax + by + cz = d$. \[5\]

May/June 2010/32

9 The plane $p$ has equation $3x + 2y + 4z = 13$. A second plane $q$ is perpendicular to $p$ and has equation $ax + y + z = 4$, where $a$ is a constant.

(i) Find the value of $a$. \[3\]

(ii) The line with equation $\mathbf{r} = \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ meets the plane $p$ at the point $A$ and the plane $q$ at the point $B$. Find the length of $AB$. \[6\]
The straight line $l$ has equation $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$. The plane $p$ has equation $3x - y + 2z = 9$. The line $l$ intersects the plane $p$ at the point $A$.

(i) Find the position vector of $A$. [3]

(ii) Find the acute angle between $l$ and $p$. [4]

(iii) Find an equation for the plane which contains $l$ and is perpendicular to $p$, giving your answer in the form $ax + by + cz = d$. [5]

### Vectors

- understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$;
- determine whether two lines are parallel, intersect or are skew;
- find the angle between two lines, and the point of intersection of two lines when it exists;
- understand the significance of all the symbols used when the equation of a plane is expressed in either of the forms $ax + by + cz = d$ or $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$;
- use equations of lines and planes to solve problems concerning distances, angles and intersections, and in particular find the equation of a line or a plane, given sufficient information, determine whether a line lies in a plane, is parallel to a plane, or intersects a plane, and find the point of intersection of a line and a plane when it exists, find the line of intersection of two non-parallel planes, find the perpendicular distance from a point to a plane, and from a point to a line, find the angle between two planes, and the angle between a line and a plane.