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1. Quadratics

1.1 Completing the square

\[ x^2 + nx \iff (x + \frac{n}{2})^2 - \left(\frac{n}{2}\right)^2 \]

Where the vertex is \((-n, k)\)

1.2 Sketching the Graph

- y-intercept
- x-intercept
- Vertex (turning point)

1.3 Discriminant

\[ b^2 - 4ac \]

- If \( b^2 - 4ac = 0 \), real and equal roots
- If \( b^2 - 4ac < 0 \), no real roots
- If \( b^2 - 4ac > 0 \), real and distinct roots

1.4 Quadratic Inequalities

\[(x - d)(x - \beta) < 0 \implies d < x < \beta\]
\[(x - d)(x - \beta) > 0 \implies x < d \text{ or } x > \beta\]

1.5 Solving Equations in Quadratic Form

- To solve an equation in some form of quadratic
- Substitute \( y \)
- E.g. \( 2x^4 + 3x^2 + 7, y = x^2 \), \( 2y^2 + 3y + 7 \)

2. Functions

- Domain = \( x \) values & Range = \( y \) values
- One-one functions: one \( x \)-value gives one \( y \)-value

2.1 Find Range

- Complete the square or differentiate
- Find min/max point
- If min then, \( y \geq \min y \)
- If max then, \( y \leq \max y \)

2.2 Composition of 2 Functions

- E.g. \( f(g(x)) \Rightarrow f(g(x)) \)

2.3 Prove One-One Functions

- One \( x \) value substitutes to give one \( y \) value
- No indices

2.4 Finding Inverse

- Make \( x \) the subject of formula

2.5 Relationship of Function & its Inverse

- The graph of the inverse of a function is the reflection of a graph of the function in \( y = x \)

(W12-P11) Question 10:

\[ f(x) = 4x^2 - 24x + 11, \text{ for } x \in \mathbb{R} \]
\[ g(x) = 4x^2 - 24x + 11, \text{ for } x \leq 1 \]

i. Express \( f(x) \) in the form \( a(x - b)^2 + c \), hence state coordinates of the vertex of the graph \( y = f(x) \)

ii. State the range of \( g \)

iii. Find an expression for \( g^{-1}(x) \) and state its domain

Solution:

Part (i)

First pull out constant, 4, from \( x \) related terms:

\[ 4(x^2 - 6x) + 11 \]

Use following formula to simplify the bracket only:

\[ \left( x - \frac{n}{2} \right)^2 = \left( \frac{n}{2} \right)^2 \]
\[ 4[(x - 3)^2 - 3^2] + 11 \]
\[ 4(x - 3)^2 - 25 \]

Part (ii)

Observe given domain, \( x \leq 1 \).

Substitute highest value of \( x \):

\[ g(x) = 4(1 - 3)^2 - 25 = -9 \]

Substitute next 3 whole numbers in domain:

\[ x = 0, -1, -2 \]  \( g(x) = 11, 23, 75 \)

Thus they are increasing

\[ : g(x) \geq -9 \]

Part (iii)

Let \( y = g(x) \), make \( x \) the subject

\[ y = 4(x - 3)^2 - 25 \]
\[ \frac{y + 25}{4} = (x - 3)^2 \]
\[ x = 3 + \sqrt{\frac{y + 25}{4}} \]

Can be simplified more

\[ x = 3 \pm \frac{1}{2}\sqrt{y + 25} \]

Positive variant is not possible because \( x \leq 1 \) and using positive variant would give values above 3

\[ \therefore x = 3 - \frac{1}{2}\sqrt{y + 25} \]
\[ \therefore g^{-1}(x) = 3 - \frac{1}{2}\sqrt{x + 25} \]

Domain of \( g^{-1}(x) = \text{Range of } g(x) \): \( x \geq -9 \)
3. COORDINATE GEOMETRY

3.1 Length of a Line Segment

Length = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}

3.2 Gradient of a Line Segment

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

3.3 Midpoint of a Line Segment

\[ \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

3.4 Equation of a Straight Line

- \[ y = mx + c \]
- \[ y - y_1 = m(x - x_1) \]

3.5 Special Gradients

- Parallel lines: \( m_1 = m_2 \)
- Perpendicular lines: \( m_1 m_2 = -1 \)
- The gradient at any point on a curve is the gradient of the tangent to the curve at that point
- The gradient of a tangent at the vertex of a curve is equal to zero – stationary point

---

**Question 10:**

Point \( R \) is a reflection of point \((-1,3)\) in the line \( 3y + 2x = 33 \).

Find by calculation the coordinates of \( R \).

**Solution:**

Find equation of line perpendicular to \( 3y + 2x = 33 \) intersecting point \((-1,3)\)

\[ 3y + 2x = 33 \Rightarrow y = 11 - \frac{2}{3}x \]

\[ m = -\frac{2}{3} \]

Perpendicular general equation:

\[ y = \frac{3}{2}x + c \]

Substitute known values

\[ 3 = \frac{3}{2}(-1) + c \] and so \( c = \frac{9}{2} \)

Final perpendicular equation:

\[ 2y = 3x + 9 \]

Find point of intersection by equating two equations

\[ 11 - \frac{2}{3}x = \frac{3x + 9}{2} \]

\[ 13 = \frac{13}{3}x \]

\[ x = 3, \quad y = 9 \]

---

4. CIRCULAR MEASURE

4.1 Radians

\( \pi = 180^\circ \) and \( 2\pi = 360^\circ \)\n
Degrees to radians: \( \times \frac{\pi}{180} \)

Radians to degrees: \( \times \frac{180}{\pi} \)

4.2 Arc length

\[ s = r\theta \]

4.3 Area of a Sector

\[ A = \frac{1}{2}r^2\theta \]

---

**Question 9:**

Triangle \( OAB \) is isosceles, \( OA = OB \) and \( ASB \) is a tangent to \( PST \)

i. Find the total area of shaded region in terms of \( r \) and \( \theta \)

ii. When \( \theta = \frac{1}{3} \) and \( r = 6 \), find total perimeter of shaded region in terms of \( \sqrt{3} \) and \( \pi \)

**Solution:**

Part (i)

Use trigonometric ratios to form the following:

\[ AS = r \tan \theta \]

Find the area of triangle \( OAS \):

\[ OAS = \frac{r \tan \theta \times r}{2} = \frac{1}{2}r^2 \tan \theta \]

Use formula of sector to find area of \( OPS \):

\[ OPS = \frac{1}{2}r^2 \theta \]

Area of \( ASP \) is \( OAS - OPS \):

\[ \therefore ASP = \frac{1}{2}r^2 \tan \theta - \frac{1}{2}r^2 \theta = \frac{1}{2}r^2(\tan \theta - \theta) \]
Multiply final by 2 because $BST$ is the same and shaded is $ASP$ and $BST$

$$Area = 2 \times \frac{1}{2} r^2 (\tan \theta - \theta) = r^2 (\tan \theta - \theta)$$

**Part (ii)**

Use trigonometric ratios to get the following:

$$\cos \left( \frac{\pi}{3} \right) = \frac{6}{AO}$$

$\therefore AO = 12$

Finding $AP$:

$$AP = AO - r = 12 - 6 = 6$$

Finding $AS$:

$$AS = 6 \tan \left( \frac{\pi}{3} \right) = 6\sqrt{3}$$

Finding arc $PS$:

$$\text{Arc } PS = r\theta$$

$$PS = 6 \times \frac{\pi}{3} = 2\pi$$

Perimeter of 1 side of the shaded region:

$$Pe_1 = 6 + 6\sqrt{3} + 2\pi$$

Perimeter of entire shaded region is just double:

$$2 \times Pe_1 = 12 + 12\sqrt{3} + 4\pi$$

### 5. Trigonometry

**5.1 Sine Curve**

Changes amplitude, increases no. of cycles, alters x-axis by $-c$

**5.2 Cosine Curve**

### 5.3 Tangent Curve

### 5.4 When sin, cos and tan are positive

### 5.5 Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$

### 6. Vectors

- **Forms of vectors**
  
  $$(\begin{array}{c}x \\ y \\ z \end{array}) = xi + yj + zk$$

- Position vector: position relative to origin $\overrightarrow{OP}$
- Magnitude: $\sqrt{x^2 + y^2}$
- Unit vectors: vectors of magnitude $1 = \frac{1}{|AB|} \overrightarrow{AB}$
- $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
- Dot product: $(ai + bj). (ci + dj) = (aci + bdj)$
- $\cos A = \frac{a.b}{|a||b|}$
Points $A, B, C, D$ have position vectors $3\mathbf{i} + 2\mathbf{j}, 2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}, 2\mathbf{j} + 7\mathbf{k}, -2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k}$ respectively.

i. Use a scalar product to show that $BA$ and $BC$ are perpendicular.

ii. Show that $BC$ and $AD$ are parallel and find the ratio of length of $BC$ to length of $AD$.

**Solution:**

First find the vectors representing $BA$ and $BC$:

$BA = OA - OB = 3\mathbf{i} + 2\mathbf{k} - (2\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

$CB = OB - OC = -2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} - (2\mathbf{j} + 7\mathbf{k}) = -2\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$

Now use the dot product rule:

$BA \cdot CB = 0$

$\begin{align*}
(1 & -2) \cdot (2 & -4) \\
3 & & -2
\end{align*}

= (-1 \times 2) + (-2 \times -4) + (3 \times -2) = 0$

Thus proving they are perpendicular since $\cos 90 = 0$

**Part (ii)**

Find the vectors representing $BC$ and $AD$:

$BC = -CB = -\begin{pmatrix} -2 \\ -4 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$

$AD = OD - OA = -2\mathbf{i} + 10\mathbf{j} + 7\mathbf{k} - (3\mathbf{i} + 2\mathbf{k}) = -5\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$

Direction vector shows that they are parallel.

Calculate lengths of each:

$|BC| = 2\sqrt{(-1)^2 + 2^2 + 1^2} = 2\sqrt{6}$

$|AD| = 5\sqrt{(-1)^2 + 2^2 + 1^2} = 5\sqrt{6}$

$\therefore |AD| : |BC| = 5 : 2$

---

**7. SERIES**

**7.1 Binomial Theorem**

$(x + y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{n}y^n$

$nC_r = \frac{n(n-1)(n-2)\ldots(n-(r-1))}{r!}$

**7.2 Arithmetic Progression**

$u_k = a + (k-1)d$

$S_n = \frac{1}{2}n[2a + (n-1)d]$

**7.3 Geometric Progression**

$u_k = ar^{k-1}$

$S_n = \frac{a(1-r^n)}{1-r}$, $S_{\infty} = \frac{a}{1-r}$

---

A small trading company made a profit of $250,000 in the year 2000. The company considered two different plans, plan $A$ and plan $B$, for increasing its profits. Under plan $A$, the annual profit would increase each year by 5% of its value in the preceding year. Under plan $B$, the annual profit would increase each year by a constant amount $D$.

i. Find for plan $A$, the profit for the year 2008

ii. Find for plan $A$, the total profit for the 10 years 2000 to 2009 inclusive

iii. Find for plan $B$ the value of $D$ for which the total profit for the 10 years 2000 to 2009 inclusive would be the same for plan $A$

**Solution:**

**Part (i)**

Increases is exponential $\because$ it is a geometric sequence: 2008 is the $9^{th}$ term:

$\therefore u_9 = 250000 \times 1.05^{9-1} = 369000$ (3.s.f.)

**Part (ii)**

Use sum of geometric sequence formula:

$S_{10} = \frac{250000(1 - 1.05^{10})}{1 - 1.05} = 3140000$

**Part (iii)**

Plan $B$ arithmetic; equate 3140000 with sum formula

$D = \frac{1}{2}(10)(2(250000) + (10 - 1)D)$

$D = 14300$
8. DIFFERENTIATION

When \( y = x^n \), \( \frac{dy}{dx} = nx^{n-1} \)

- 1st Derivative: \( f'(x) \)
- 2nd Derivative: \( f''(x) \)
- Increasing function: \( \frac{dy}{dx} > 0 \)
- Decreasing function: \( \frac{dy}{dx} < 0 \)
- Stationary point: \( \frac{dy}{dx} = 0 \)

8.1 Chain Rule

\[
\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}
\]

8.2 Nature of Stationary Point

- Find second derivative
- Substitute \( x \)-value of stationary point
- If value +ve \( \rightarrow \) min. point
- If value -ve \( \rightarrow \) max. point

8.3 Connected Rates of Change

\[
\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}
\]

### Question 6:

The equation of a curve is given by the formula:

\[
y = \frac{6}{5 - 2x}
\]

**i.** Calculate the gradient of the curve at the point where \( x = 1 \)

**ii.** A point with coordinates \((x, y)\) moves along a curve in such a way that the rate of increase of \( y \) has a constant value of 0.02 units per second. Find the rate of increase of \( x \) when \( x = 1 \)

**Solution:**

**Part (i)**

Differentiate given equation

\[
6(5 - 2x)^{-1}
\]

\[
\frac{dy}{dx} = 6(5 - 2x)^{-2} \times -2x - 1
\]

\[
= 12(5 - 2x)^{-2}
\]

Now we substitute the given \( x \)-value:

\[
\frac{dy}{dx} = 12(5 - 2(1))^{-2}
\]

\[
\frac{dy}{dx} = \frac{4}{3}
\]

Thus the gradient is equal to \( \frac{1}{3} \) at this point

**Part (ii)**

Rate of increase in time can be written as:

\[
\frac{dx}{dt}
\]

We know the following:

\[
\frac{dy}{dx} = \frac{4}{3} \quad \text{and} \quad \frac{dy}{dt} = 0.02
\]

Thus we can formulate an equation:

\[
\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dy}{dt}
\]

Rearranging the formula we get:

\[
\frac{dx}{dt} = \frac{dy}{dt} \div \frac{dy}{dx}
\]

Substitute values into the formula

\[
\frac{dx}{dt} = 0.02 \div \frac{4}{3}
\]

\[
\frac{dx}{dt} = 0.02 \times \frac{3}{4} = 0.015
\]

9. INTEGRATION

\[
\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c
\]

\[
\int (ax + b)^n = \frac{(ax + b)^{n+1}}{a(n+1)} + c
\]

Definite integrals: substitute coordinates and find ‘c’

9.1 To Find Area

- Integrate curve
- Substitute boundaries of \( x \)
- Subtract one from another (ignore c)

\[
\int_c^d y \, dx
\]

9.2 To Find Volume

- Square the function
- Integrate and substitute
- Multiply by \( \pi \)

\[
\int_c^d \pi y^2 \, dx
\]