Q1.

7 (i) State \( \frac{dx}{d\theta} = 2 - 2\cos 2\theta \) or \( \frac{dy}{d\theta} = 2\sin 2\theta \)

Use \( \frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} \)

Obtain answer \( \frac{dy}{dx} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta} \) or equivalent

Make relevant use of \( \sin 2\theta \) and \( \cos 2\theta \) formulae (indep.)

Obtain given answer correctly

[5]

(ii) Substitute \( \theta = \frac{1}{4} \pi \) in \( \frac{dy}{dx} \) and both parametric equations

Obtain \( \frac{dy}{dx} = 1, x = \frac{1}{2} \pi - 1, y = 2 \)

Obtain equation \( y = x + 1.43 \), or any exact equivalent

[3]

(iii) State or imply that tangent is horizontal when \( \theta = \frac{1}{2} \pi \) or \( \frac{3}{2} \pi \)

Obtain a correct pair of \( x \), \( y \) or \( x \)-or \( y \)-coordinates

State correct answers \( (\pi, 3) \) and \( (3\pi, 3) \)

[3]

Q2.

6 (i) State that \( \frac{dx}{dt} = 2 + \frac{1}{t} \) or \( \frac{dy}{dt} = 1 - \frac{4}{t^2} \), or equivalent

Use \( \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} \)

Obtain the given answer

[3]

(ii) Substitute \( t = 1 \) in \( \frac{dy}{dx} \) and both parametric equations

Obtain \( \frac{dy}{dx} = -1 \) and coordinates \( (2, 5) \)

State equation of tangent in any correct horizontal form e.g. \( x + y = 7 \)

[3]

(iii) Equate \( \frac{dy}{dx} \) to zero and solve for \( t \)

Obtain answer \( t = 2 \)

Obtain answer \( y = 4 \)

Show by any method (but not via \( \frac{d}{dt} (y') \)) that this is a minimum point

[4]
Q3.

5 (i) Differentiate using chain or quotient rule
    Obtain derivative in any correct form
    Obtain given answer correctly

(ii) State \( \frac{dx}{d\theta} = \sec^2 \theta \), or equivalent
    Use \( \frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} \)
    Obtain given answer correctly

Q4.

3 State correct derivative \( 1 - 2\sin x \)
   Equate derivative to zero and solve for \( x \)
   Obtain answer \( x = \frac{\pi}{6} \)
   Carry out an appropriate method for determining the nature of a stationary point
   Show that \( x = \frac{\pi}{6} \) is a maximum with no errors seen
   Obtain second answer \( x = \frac{\pi}{3} \) in range
   Show thus is a minimum point
   [if \( x \) is on the incorrect derivative \( 1 + 2\sin x \)]

Q5.
Q6.

3 (i) State \( \frac{dx}{dt} = 3 + \frac{1}{t-1} \) or \( \frac{dy}{dt} = 2t \)

Use \( \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \)

Obtain \( \frac{dy}{dx} \) in any correct form, e.g., \( \frac{2t(t-1)}{3t-2} \)

(ii) Equate derivative to 1 and solve for \( t \)

Obtain roots 2 and \( \frac{1}{2} \)

State or imply that only \( t = 2 \) is admissible c.w.o.

Obtain coordinates (6, 5)

Q7.

6 (i) Use product rule

Obtain correct derivative in any form, e.g., \( (x-1)e^t \)

Equate derivative to zero and solve for \( x \)

Obtain \( x = 1 \)

Obtain \( y = -e \)

(ii) Carry out a method for determining the nature of a stationary point

Show that the point is a minimum point, with no errors seen

Q8.
Q9.

4 State \( \frac{dx}{d\theta} = 4 \cos \theta \)
   \( \frac{dy}{d\theta} = 4 \sin 2\theta \), or equivalent
   Use \( \frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx} \)
   Obtain \( \frac{dy}{dx} \) in any correct form, e.g. \( \frac{\sin 2\theta}{\cos \theta} \)
   Simplify and obtain answer \( 2 \sin \theta \)
   \( [\text{The f.t. is on gradients of the form } k \sin 2\theta / \cos \theta, \text{ or equivalent.}] \)

Q10.

6 (i) State \( 2xy + x^2 \frac{dy}{dx} \) as derivative of \( x^2y \)
   \( 2y \frac{dy}{dx} \) as derivative of \( y^2 \)
   Equate derivatives of LHS and RHS, and solve for \( \frac{dy}{dx} \)
   Obtain given answer
   \( [\text{The M1 is dependent on at least one of the B marks being obtained.}] \)

(ii) Substitute and obtain gradient \( \frac{2}{3} \), or equivalent
    Form equation of tangent at the given point \((1, 2)\)
    Obtain answer \( 2x - 5y + 8 = 0 \), or equivalent
    \( [\text{The M1 is dependent on at least one of the B marks being obtained.}] \)

Q11.
Q12.

2 State \( \frac{dx}{dt} = 3 + 2 \cos 2t \) or \( \frac{dy}{dt} = -4 \sin 2t \) (or both) \( \text{B1} \)

Use \( \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} \) \( \text{M1} \)

Obtain or imply \( \frac{-4 \sin 2t}{3 + 2 \cos 2t} \) \( \text{A1} \)

Substitute \( \frac{1}{6} \pi \) to obtain \( \frac{1}{2} \sqrt{3} \) or exact equivalent \( \text{A1} [4] \)

Q13.

5 (i) Differentiate \( \ln(x - 3) \) to obtain \( \frac{1}{x - 3} \) \( \text{B1} \)

Attempt to use product rule \( \text{M1} \)

Obtain \( \ln(x - 3) + \frac{x}{x - 3} \) or equivalent \( \text{A1} \)

Substitute 4 to obtain \( 4 \) \( \text{A1} [4] \)

(ii) Use correct quotient or product rule \( \text{M1} \)

Obtain correct derivative in any form, e.g. \( \frac{(x+1)-(x-1)}{(x+1)^2} \) \( \text{A1} \)

Substitute 4 to obtain \( \frac{2}{25} \) \( \text{A1} [3] \)

Q14.

5 Obtain \( 4y \frac{dy}{dx} \) as derivative of \( 2y^2 \) \( \text{B1} \)

Differentiate LHS term by term to obtain expression including at least one \( \frac{dy}{dx} \) \( \text{M1} \)

Obtain \( 2x + 4y \frac{dy}{dx} + 5 + 6 \frac{dy}{dx} \) \( \text{A1} \)

Substitute 2 and \( -1 \) to attempt value of \( \frac{dy}{dx} \) \( \text{M1} \)

Obtain \( -\frac{9}{2} \) \( \text{A1} \)

Obtain equation \( 9x + 2y - 16 = 0 \) or equivalent of required form \( \text{A1} [6] \)
Q15.

6 (i) Attempt differentiation using product rule
   Obtain \( 8x \ln x + 4x \) (a.c.f.)
   Equate first derivative to zero and attempt solution
   Obtain 0.607
   Obtain -0.736 following their \( x \)-coordinate
   A1\[5\]

   (ii) Use an appropriate method for determining nature of stationary point
        Conclude point is a minimum (with no errors seen, second derivative = 8)
        A1\[2\]

Q16.

5 (i) State \( \frac{dx}{dt} = \frac{1}{t+1} \)
   State \( \frac{dy}{dt} = 2e^{2t} + 2 \)
   Attempt expression for \( \frac{dy}{dx} \)
   Obtain \( \frac{dy}{dx} = \frac{(2e^{2t} + 2)(t + 1)}{t+1} \) or equivalent
   B1\[4\]

   (ii) Substitute \( t = 0 \) and attempt gradient of normal
        Obtain \( -\frac{1}{2} \) following their expression for \( \frac{dy}{dx} \)
        Attempt to find equation of normal through point \((0, 1)\)
        Obtain \( x + 4y - 4 = 0 \)
        A1\[4\]

Q17.

5 (i) Use product rule to differentiate \( y \)
   Obtain correct derivative in any form
   Use \( \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dx}{dt} \)
   Obtain given answer correctly
   A1\[4\]

   (ii) Substitute \( t = 0 \) in \( \frac{dy}{dx} \) and both parametric equations
        Obtain \( \frac{dy}{dx} = 2 \) and coordinates \((1, 0)\)
        Form equation of the normal at their point, using negative reciprocal of their \( \frac{dy}{dx} \)
        State correct equation of normal \( y = -\frac{1}{2}x + \frac{1}{2} \) or equivalent
        B1\[4\]

Q18.
Q19.

5 (i) State \( \frac{dy}{dx} \) as derivative of \( 3y \), or equivalent
State \( 4xy + 2x^2 \frac{dy}{dx} \) as a derivative of \( 2x^3y \), or equivalent
Equate derivative of LHS to zero and solve for \( \frac{dy}{dx} \)
Obtain given answer correctly

(ii) Substitute \( x = 2 \) into given equation and solve for \( y \)
Obtain gradient = \( \frac{12}{5} \) correctly
Form equation of the normal at their point, using negative recip of their \( \frac{dy}{dx} \)
State correct equation of normal \( 5x + 12y + 2 = 0 \) or equivalent

Q20.
Q21.
5 (i) Use the product rule to obtain the first derivative (must involve 2 terms) M1
Obtain derivative \(2x \ln x + x^2 \frac{1}{x}\) or equivalent A1
Equate derivative to zero and solve for \(x\) M1
Obtain answer \(x = e^{-0.5}\) or \(\frac{1}{\sqrt{e}}\) or equivalent (e.g. 0.61) A1

(ii) Determine nature of stationary point using correct second derivative (or correct first derivative or equation of the curve (3 \(y\)-values, central one \(y(\exp (-0.5))\)) M1
Show point is a minimum completely correctly A1

Q22.
4 (i) State \(3y^2 \frac{dy}{dx}\) as derivative of \(y^3\) B1
State \(9y - 9x \frac{dy}{dx}\) as derivative of \(9xy\) B1
Express \(\frac{dy}{dx}\) in terms of \(x\) and \(y\) M1
Obtain given answer correctly A1
(The M1 is conditional on at least one B mark being obtained.)

(ii) Obtain gradient at (2, 4) in any correct unsimplified form B1
Form the equation of the tangent at (2, 4) M1
Obtain answer \(5y - 4x = 12\), or equivalent A1
Q23.

4 State derivative \(2 - \sec^2 x\), or equivalent

Equate derivative to zero and solve for \(x\)

Obtain \(x = \frac{1}{4} \pi\), or 0.785 (± 45° gains A1)

Obtain \(x = -\frac{1}{4} \pi\), (allow negative of first solution) A1 √

Obtain corresponding \(y\)-values \(\frac{1}{2} \pi - 1\) and \(\frac{1}{2} \pi + 1\), ± 0.571 A1 [5]

Q24.

6 At any stage, state the correct derivative of \(e^{\frac{1}{2}x}\) or \(e^{\frac{1}{2}x}\)

Use product or quotient rule B1

Obtain correct first derivative in any form M1

Obtain correct second derivative in any form B1 √

Equate second derivative to zero and solve for \(x\)

Obtain \(x = 4\) A1

Obtain \(y = 4e^{-2}\), or equivalent A1 [7]

Q25.

6 (i) Use product rule M1*

Obtain derivative in any correct form A1

Equate derivative to zero and solve for \(x\) M1 (dep*)

Obtain \(x = 1/e\), or exact equivalent A1

Obtain \(y = -1/e\), or exact equivalent A1 [5]

(ii) Carry out complete method for determining the nature of a stationary point M1

Show that at \(x = 1/e\) there is a minimum point, with no errors seen A1 [2]

Q26.

8 (i) EITHER: Substitute \(x = 1\) and attempt to solve 3-term quadratic in \(y\) M1

Obtain answers (1, 1) and (1, -3) A1

OR: State answers (1, 1) and (1, -3) B1 + B1 [2]

(ii) State \(2y \frac{dy}{dx}\) as derivative of \(y^2\) B1

State \(2y + 2x \frac{dy}{dx}\) as derivative of \(2xy\) B1

Substitute for \(x\) and \(y\), and solve for \(\frac{dy}{dx}\) M1

Obtain \(\frac{dy}{dx} = 0\) when \(x = 1\) and \(y = 1\) A1

Obtain \(\frac{dy}{dx} = -2\) when \(x = 1\) and \(y = -3\) A1 √

Form the equation of the tangent at (1, -3) M1

Obtain answer \(2x + y + 1 = 0\) A1 [7]
Q27.

4 (i) State \( \frac{dx}{dt} = e^t \) or \( \frac{dy}{dt} = e^t - e^{-t} \) \( \text{B1} \)

Use \( \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} \) \( \text{M1} \)

Obtain given answer correctly \( \text{A1} \) [3]

(ii) Substitute \( \frac{dy}{dx} = 2 \) and use correct method for solving an equation of the form \( e^{2t} = a \),

where \( a > 0 \) \( \text{M1} \)

Obtain answer \( t = \frac{1}{2} \ln 3 \), or equivalent \( \text{A1} \) [2]

Q28.

4 (i) State \( \frac{dx}{dt} = \frac{1}{t - 2} \) or \( \frac{dy}{dt} = 1 - 9t^2 \) \( \text{B1} \)

Use \( \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} \) \( \text{M1} \)

Obtain given answer correctly \( \text{A1} \) [3]

(ii) Equate derivative to zero and solve for \( t \)

State or imply that \( t = 3 \) is admissible c.w.o., and note \( t = -3 \), 2 cases \( \text{A1} \)

Obtain coordinates \((1, 6)\) and no others \( \text{A1} \) [3]

Q29.

8 (i) State \( 2y \frac{dy}{dx} \) as derivative of \( y^2 \), or equivalent \( \text{B1} \)

State \( 2y + 2x \frac{dy}{dx} \) as derivative of \( 2xy \), or equivalent \( \text{B1} \)

Substitute \( x = -2 \) and \( y = 2 \) and evaluate \( \frac{dy}{dx} \) \( \text{M1} \)

Obtain zero correctly and make correct conclusion \( \text{A1} \) [4]

(ii) Substitute \( x = -2 \) into given equation and solve \( \text{M1} \)

Obtain \( y = -6 \) correctly \( \text{A1} \)

Obtain \( \frac{dy}{dx} = 2 \) correctly \( \text{B1} \)

Form the equation of the tangent at \((-2, -6)\) \( \text{M1} \)

Obtain answer \( y = 2x - 2 \) \( \text{A1} \) [5]

Q30.
Q31.

7  (i) Use product rule to differentiate $y$
   Obtain correct derivative in any form in $t$ for $y$
   Use $\frac{dy}{dx} = \frac{dy}{dt} + \frac{dx}{dt}$
   Obtain given answer correctly
   (ii) State $t = 0$
   State that $\frac{dy}{dx} = 0$ and make correct conclusion
   (iii) Substitute $t = -2$ into equation for $x$ or $y$
   Obtain $(e^x, 4e^{-2} + 3)$

Q32.

6  (i) State $\frac{dx}{d\theta} = 4\sin\theta\cos\theta$ or equivalent (nothing for $\frac{dy}{dx} = 4\sec^2\theta$)
   Use $\frac{dy}{dx} = \frac{dy}{d\theta} + \frac{dx}{d\theta}$
   Obtain given answer correctly
   (ii) Substitute $\theta = \frac{\pi}{4}$ in $\frac{dy}{dx}$ and both parametric equations
   Obtain $\frac{dy}{dx} = 4$ and coordinates $(2, 4)$
   Form equation of tangent at their point
   State equation of tangent in correct form $y = 4x - 4$

Q33.

1  Obtain derivative of the form $\frac{k}{5x+1}$, where $k = 1, 5$ or $\frac{1}{5}$
   Obtain correct derivative $\frac{5}{5x+1}$
   Substitute $x = 4$ into expression for derivative and obtain $\frac{5}{21}$

Q34.
Q35.

8 (i) State \( 2y \frac{dy}{dx} \) as derivative of \( y^2 \), or equivalent

\[
\text{Equate derivative of LHS to zero and solve for } \frac{dy}{dx}
\]

Obtain given answer correctly

M1 A1 [3]

(ii) Equate gradient expression to \(-1\) and rearrange

Obtain \( y = 2x \)

Substitute into original equation to obtain an equation in \( x^2 \) (or \( y^2 \))

Obtain \( 2x^2 - 3x - 2 = 0 \) (or \( y^2 - 3y - 4 = 0 \))

Correct method to solve their quadratic equation

State answers \((-\frac{1}{2}, -1)\) and \((2, 4)\)


Q36.

4 (i) State \( \frac{dx}{dt} = \frac{-2}{1 - 2t} \) or \( \frac{dy}{dt} = -2t^{-2} \)

Use \( \frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \)

Obtain given answer correctly

M1 A1 [3]

(ii) Equate derivative to \(3\) and solve for \(t\)

State or imply that \(t = -1\) c.w.o.

Obtain coordinates \((\ln 3, -2)\)

M1 A1 [3]

Q37.

2 Use quotient rule or product rule, correctly

Obtain correct derivative in any form

Equate derivative to zero and solve for \(x\)

Obtain \( x = \frac{\pi}{8} \)

M1 A1 [4]
Q38.

3 Obtain derivative $e^{2x} - 5e^x + 4$
Equate derivative to zero and carry out recognisable solution method for a quadratic in $e^x$
Obtain $e^x = 1$ or $e^x = 4$
Obtain $x = 0$ and $x = \ln 4$
Use an appropriate method for determining nature of at least one stationary point
\[
\left(\frac{d^2y}{dx^2} = 2e^{2x} - 5e^x, \text{ when } x = 0, \frac{d^2y}{dx^2} = -(3), x = \ln 4, \frac{d^2y}{dx^2} = +(12)\right)
\]
Conclude maximum at $x = 0$ and minimum at $x = \ln 4$ (no errors seen)

Q39.

5 (i) State $\frac{dx}{d\theta} = -2\sin 2\theta + \sin \theta$ or $\frac{dy}{d\theta} = 8\sin \theta \cos \theta$
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$
Use $\sin 2\theta = 2\sin \theta \cos \theta$
Obtain given answer correctly
(ii) Equate derivative to $-4$ and solve for $\cos \theta$
Obtain $\cos \theta = \frac{1}{2}$
Obtain $x = -1$
Obtain $y = 3$

Q40.

2 Use quotient or product rule
Obtain correct derivative in any form
Equate (numerator) of derivative to zero and solve for $x$
Obtain $x = \frac{1}{3}$
Obtain $y = \frac{1}{2}$

Q41.
### Q42.

2. (i) Differentiate to obtain form \( k_1 \cos x + k_2 \sec^2 2x \)
   - Obtain correct second term \( 2\sec^2 2x \)
   - Obtain \( 3\cos x + 2\sec^2 2x \) and hence answer 5

   \[ \text{M1} \]

   \[ \text{A1} \] \[3\]

(ii) Differentiate to obtain form \( ke^{2z}(1+e^{2z})^2 \)
   - Obtain correct \(-12e^{2z}(1+e^{2z})^2 \) or equivalent (may be implied)
   - Obtain \(-3\)

   \[ \text{M1} \]

   \[ \text{A1} \] \[3\]

### Q43.

7. (i) Obtain \( 3y + 3x \frac{dy}{dx} \) as derivative of \( 3xy \)
   - Obtain \( 2y \frac{dy}{dx} \) as derivative of \( y^2 \)
   - State \( 4x + 3y + 3x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0 \)
   - Substitute 2 and \(-1\) to find gradient of curve (dependent on at least one \( \text{B1} \))
   - Form equation of tangent through \((2, -1)\) with numerical gradient
     (dependent on previous \( \text{M1} \))
   - Obtain \( 5x + 4y - 6 = 0 \) or equivalent of required form

   \[ \text{B1} \]

   \[ \text{B1} \]

   \[ \text{B1} \]

   \[ \text{M1} \]

   \[ \text{DM1} \]

   \[ \text{A1} \] \[6\]

(ii) Use \( \frac{dy}{dx} = 0 \) to find relation between \( x \) and \( y \)
   - (dependent on at least one \( \text{B1} \) from part(i))
   - Obtain \( 4x + 3y = 0 \) or equivalent
   - Substitute for \( x \) or \( y \) in equation of curve
   - Obtain \(- \frac{1}{2} y^2 = 3 \) or \(- \frac{2}{3} x^2 = 3 \) or equivalent and conclude appropriately

   \[ \text{M1} \]

   \[ \text{A1} \]

   \[ \text{A1} \] \[4\]

### Q44.
4. \( \frac{dx}{dr} = \frac{2}{t+1} \)  

Obtain \( \frac{dy}{dt} = 4e^t \)  

Use \( \frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} \) with \( t = 0 \) to find gradient  

Obtain 2  

Form equation of tangent through \((0, 4)\) with numerical gradient obtained from attempt to differentiate  

Obtain \( 2x - y + 4 = 0 \) or equivalent of required form  

Q45.

8. (i) Differentiate using product rule  
Obtain \( \sec^3x \cos 2x - 2 \tan x \sin 2x \)  

Use \( \cos 2x = 2 \cos^2 x - 1 \) or \( \sin 2x = 2 \sin x \cos x \) or both  

Express derivative in terms of \( \sec x \) and \( \cos x \) only  

Obtain \( 4 \cos^3 x - \sec^2 x - 2 \) with no errors seen (AG)  

(ii) State \( 4 \cos^2 x - 2 \cos^2 x - 1 = 0 \)  

Apply quadratic formula to a 3 term quadratic equation in terms of \( \cos^2 x \) to find the least positive value of \( \cos^2 x \)  

Obtain or imply \( \cos^2 x = \frac{1 + \sqrt{5}}{4} \) or 0.809...  

Obtain 0.45  

Q46.

4. (i) Differentiate to obtain form \( k_1 \sin 2x + k_2 \cos x \)  
Obtain correct \( -6 \sin 2x - 5 \cos x \)  

Substitute \( \frac{1}{6} \pi \) to obtain \( -\frac{11}{2} \sqrt{3} \) or exact equivalent  

(ii) Obtain \( 6y + 6x \frac{dy}{dx} \) as derivative of \( 6xy \)  

Obtain \( 3y^3 \frac{dy}{dx} \) as derivative of \( y^3 \)  

Obtain \( 3x^2 + 6y + 6x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0 \) or equivalent  

Substitute 1 and 2 to find value of gradient dependent on at least one  

Obtain gradient \( -\frac{15}{18} \) or \( -\frac{5}{6} \)  

Q47.
P3 (variant1 and 3)

Q1.

9 (i) Use quotient or product rule to differentiate \((1-x)/(1+x)\)

- Obtain correct derivative in any form
- Use chain rule to find \(dy/dx\)
- Obtain a correct expression in any form
- Obtain the gradient of the normal in the given form correctly

(ii) Use product rule

- Obtain correct derivative in any form
- Equate derivative to zero and solve for \(x\)

Q2.

2 (i) Obtain \(\frac{k \cos 2x}{1+\sin 2x}\) for any non-zero constant \(k\)

- Obtain \(\frac{2 \cos 2x}{1+\sin 2x}\)

(ii) Use correct quotient or product rule

- Obtain \(\frac{x \sec^2 x - \tan x}{x^2}\) or equivalent

Q3.
Q4.

2. Use correct quotient or product rule

Obtain correct derivative in any form, e.g. \( \frac{\ln x}{x^5} + \frac{1}{x^4} \)

Equate derivative to zero and solve for \( x \) an equation of the form \( \ln x = a \), where \( a > 0 \)

Obtain answer \( \exp\left(\frac{1}{3}\right) \), or 1.40, from correct work

Q5.

6. (i) Obtain \( 2y \cdot \frac{dy}{dx} \) as derivative of \( y^2 \)

Obtain \(-4y - 4x \cdot \frac{dy}{dx}\) as derivative of \(-4xy\)

Substitute \( x = 2 \) and \( y = -3 \) and find value of \( \frac{dy}{dx} \)

(dependent on at least one B1 being earned and \( \frac{d(45)}{dx} = 0 \))

Obtain \( \frac{12}{7} \) or equivalent

(ii) Substitute \( \frac{dy}{dx} = 1 \) in an expression involving \( \frac{dy}{dx}, x \) and \( y \) and obtain \( ay = bx \)

Obtain \( y = x \) or equivalent

Uses \( y = x \) in original equation and demonstrate contradiction

Q6.
Q7.

4 (i) Use correct quotient or product rule

Obtain correct derivative in any form, e.g. \( \frac{2e^{2x}}{x^3} - \frac{3e^{2x}}{x^4} \)

Equate derivative to zero and solve a 2-term equation for non-zero \( x \)

Obtain \( x = \frac{3}{2} \) correctly

(ii) Carry out a method for determining the nature of a stationary point, e.g. test derivative either side

Show point is a minimum with no errors seen

Q8.

5 (i) Use correct quotient rule or equivalent

Obtain \( \frac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2} \) or equivalent

Substitute \( x = 0 \) and obtain \( -\frac{1}{2} \) or equivalent

(ii) Differentiate \( y^3 \) and obtain \( 3y^2 \frac{dy}{dx} \)

Differentiate \( 5xy \) and obtain \( 5y + 5x \frac{dy}{dx} \)

Obtain \( 6x^2 + 5y + 5x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0 \)

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<td>GCE AS/A LEVEL – May/June 2013</td>
<td>9709</td>
<td>31</td>
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Substitute \( x = 0, y = 2 \) to obtain \( -\frac{5}{6} \) or equivalent following correct work
Q9.

4 Use product or quotient rule
Obtain derivative in any correct form
Equate derivative to zero and obtain an equation of the form \(a \sin 2x = b\), or a quadratic in \(\tan x\), \(\sin^2 x\), or \(\cos^2 x\)
M1*
Carry out correct method for finding one angle
M1 (dep*)
Obtain answer, e.g. 0.365
A1
Obtain second answer 1.206 and no others in the range (allow 1.21)
[Ignore answers outside the given range.]
[ Treat answers in degrees, 20.9° and 69.1°, as a read.] A1 [6]

Q10.

2 Use of correct quotient or product rule to differentiate \(x\) or \(t\)
Obtain correct \(\frac{3}{(2t + 3)^2}\) or unsimplified equivalent
A1
Obtain \(-2e^{2x}\) for derivative of \(y\)
B1
Use \(\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}\) or equivalent
M1
cwo A1 [5]

**Alternative:**
Eliminate parameter and attempt differentiation \(\left( y = e^{\frac{4x}{1-2x}} \right) \)
B1
Use correct quotient or product rule
M1
Use chain rule
M1
Obtain \(\frac{dy}{dx} = \frac{-6}{(1-2x)^2} e^{\frac{4x}{1-2x}}\)
A1
Obtain \(-6\)
cwo A1

Q11.

2 **EITHER:** Use chain rule
obtain \(\frac{dx}{dt} = 6 \sin t \cos t\), or equivalent
A1
obtain \(\frac{dy}{dt} = -6 \cos^2 t \sin t\), or equivalent
A1
Use \(\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}\)
M1
Obtain final answer \(\frac{dy}{dx} = -\cos t\)
A1

**OR:**
Express \(y\) in terms of \(x\) and use chain rule
M1
Obtain \(\frac{dy}{dx} = k(2 - \frac{x}{3})^\frac{1}{2}\), or equivalent
A1
Obtain \(\frac{dy}{dx} = -(2 - \frac{x}{3})^\frac{1}{2}\), or equivalent
A1
Express derivative in terms of \(t\)
M1
Obtain final answer \(\frac{dy}{dx} = -\cos t\)
Q12.

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<tr>
<td>2</td>
<td>Use correct quotient or product rule or equivalent</td>
<td>M1</td>
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<td>Obtain ( \frac{(1+e^{2x})2e^{2x} - e^{2x}2e^{2x}}{(1+e^{2x})^2} ) or equivalent</td>
<td>A1</td>
<td></td>
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<td></td>
<td>Substitute ( x = \ln 3 ) into attempt at first derivative and show use of relevant logarithm property at least once in a correct context</td>
<td>M1</td>
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<td>Confirm given answer ( \frac{3}{50} ) legitimately</td>
<td>A1</td>
<td>[4]</td>
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Q13.

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<tr>
<td>8</td>
<td>(i) Differentiate ( y ) to obtain ( 3\sin^2 t \cos t - 3\cos^2 t \sin t ) t o.e.</td>
<td>B1</td>
<td></td>
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<td>Use ( \frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt} )</td>
<td>M1</td>
<td></td>
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<td></td>
<td>Obtain given result (-3\sin t \cos t )</td>
<td>A1cwo</td>
<td>[3]</td>
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<td>(ii) Identify parameter at origin as ( t = \frac{3}{4} \pi )</td>
<td>B1</td>
<td></td>
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<tr>
<td></td>
<td>Use ( t = \frac{3}{4} \pi ) to obtain ( \frac{3}{2} )</td>
<td>B1</td>
<td>[2]</td>
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<td>(iii) Rewrite equation as equation in one trig variable ( e.g. \sin 2t = -\frac{1}{2}, 9 \sin^4 x - 9 \sin^2 x + 1 = 0, \tan^2 x + 3 \tan x + 1 = 0 )</td>
<td>B1</td>
<td></td>
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<tr>
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<td>Find at least one value of ( t ) from equation of form ( \sin 2t = k ) o.e.</td>
<td>M1</td>
<td></td>
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<td>Obtain 1.9</td>
<td>A1</td>
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<td>Obtain 2.8 and no others</td>
<td>A1</td>
<td>[4]</td>
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Q14.
Q15.

3 (i) Either Use correct quotient rule or equivalent to obtain
\[
\frac{dy}{dt} = \frac{4(2t+3) - 8t}{(2t+3)^2} \quad \text{or equivalent}
\]
Obtain \( \frac{dy}{dt} = \frac{4}{2t+3} \) \( \text{or equivalent} \)
Use \( \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \) \( \text{or equivalent} \)
Obtain \( \frac{1}{3} (2t+3) \) or similarly simplified equivalent

Or
Express \( t \) in terms of \( x \) or \( y \) e.g. \( t = \frac{3x}{4-2x} \)
Obtain Cartesian equation e.g. \( y = 2 \ln \left( \frac{6}{2-x} \right) \)
Differentiate and obtain \( \frac{dy}{dx} = -\frac{2}{2-x} \)
Obtain \( \frac{1}{3} (2t+3) \) or similarly simplified equivalent

(ii) Obtain \( 2t = 3 \) or \( t = \frac{3}{2} \)
Substitute in expression for \( \frac{dy}{dx} \) and obtain 2

Q16.

1 Use correct quotient or product rule
Obtain correct derivative in any form
Justify the given statement
Q17.

4 Use correct product or quotient rule at least once

\[
\frac{dx}{dt} = e^{-t} \sin t - e^{-t} \cos t \quad \text{or} \quad \frac{dy}{dt} = e^{-t} \cos t - e^{-t} \sin t, \quad \text{or equivalent}
\]

Use \( \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} \) \( \text{M1} \)

Obtain \( \frac{dy}{dx} = \frac{\sin t - \cos t}{\sin t + \cos t} \), or equivalent

\( \text{A1} \)

\( \text{EITHER: Express } \frac{dy}{dx} \text{ in terms of } \tan t \text{ only} \)

Show expression is identical to \( \tan \left( t - \frac{1}{4} \pi \right) \)

\( \text{A1} \)

\( \text{OR: Express } \tan \left( t - \frac{1}{4} \pi \right) \text{ in terms of } \tan t \)

\( \text{M1} \)

Show expression is identical to \( \frac{dy}{dx} \)

\( \text{A1} \) [6]

Q18.

4 Differentiate \( y^3 \) to obtain \( 3y^2 \frac{dy}{dx} \)

Use correct product rule at least once

\( \text{B1} \)

Obtain \( 6e^{2t}y + 3e^{2t} \frac{dy}{dx} + e^{t}y^3 + 3e^{t}y^2 \frac{dy}{dx} \) as derivative of LHS

\( \text{A1} \)

Equate derivative of LHS to zero, substitute \( x = 0 \) and \( y = 2 \) and find value of \( \frac{dy}{dx} \)

\( \text{M1(d*M)} \)

Obtain \(-\frac{4}{3}\) or equivalent as final answer

\( \text{A1} \) [5]

Q19.

3 Obtain \( \frac{2}{2t+3} \) for derivative of \( x \)

Use quotient of product rule, or equivalent, for derivative of \( y \)

\( \text{M1} \)

Obtain \( \frac{5}{(2t+3)^2} \) or unsimplified equivalent

\( \text{A1} \)

Obtain \( t = -1 \)

\( \text{B1} \)

Use \( \frac{dy}{dx} = \frac{dy}{dr} \frac{dr}{dx} \) in algebraic or numerical form

\( \text{M1} \)

Obtain gradient \( \frac{5}{2} \)

\( \text{A1} \) [6]

Q20.
Q21.

5 Obtain correct derivative of RHS in any form
   Obtain correct derivative of LHS in any form
   Set \( \frac{dy}{dx} \) equal to zero and obtain a horizontal equation
   Obtain a correct equation, e.g. \( x^2 + y^2 = 1 \), from correct work
   By substitution in the curve equation, or otherwise, obtain an equation in \( x^2 \) or \( y^2 \)
   Obtain \( x = \frac{1}{2} \sqrt{3} \)
   Obtain \( y = \frac{1}{2} \)

Q22.

4 (i) Use chain rule correctly at least once
    Obtain either \( \frac{dx}{dt} = \frac{3\sin t}{\cos t} \) or \( \frac{dy}{dt} = 3\tan^2 \sec^2 t \), or equivalent
    Use \( \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \)
    Obtain the given answer

(ii) State a correct equation for the tangent in any form
    Use Pythagoras
    Obtain the given answer

Q23.
2. Use correct product rule or correct chain rule to differentiate \( y \)

Use \( \frac{dy}{dx} = \frac{dy}{d\theta} \)

\[ A1 \]

\[ M1 \]

Obtain \( \frac{-4 \cos \theta \sin^2 \theta + 2 \cos^2 \theta}{\sec^2 \theta} \) or equivalent

Express \( \frac{dy}{dx} \) in terms of \( \cos \theta \)

\[ DM*1 \]

Confirm given answer \( 6 \cos^3 \theta - 4 \cos^3 \theta \) legitimately

\[ A1 \] [5]
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