These are P2 questions(all variants) as the syllabus is same as P3 :)

Q1.

The diagram shows the curve $y = e^{2x}$. The shaded region $R$ is bounded by the curve and by the lines $x = 0$, $y = 0$ and $x = p$.

(i) Find, in terms of $p$, the area of $R$.  

(ii) Hence calculate the value of $p$ for which the area of $R$ is equal to 5. Give your answer correct to 2 significant figures.

Q2.

7 (i) By expanding $\cos(2x + x)$, show that

$$\cos 3x = 4 \cos^3 x - 3 \cos x.$$  

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{\pi}{4}} \cos^3 x \, dx = \frac{\pi}{3}.$$  

Q3.

7 (i) By expanding $\sin(2x + x)$ and using double-angle formulae, show that

$$\sin 3x = 3 \sin x - 4 \sin^3 x.$$  

(ii) Hence show that

$$\int_0^{\frac{\pi}{4}} \sin^3 x \, dx = \frac{5}{24}.$$
Q4.

6  (i) Express \( \cos^2 x \) in terms of \( \cos 2x \). \[1\]

(ii) Hence show that

\[
\int_{0}^{\frac{3\pi}{2}} \cos^2 x \, dx = \frac{1}{8} \pi + \frac{1}{8} \sqrt{3}.
\] \[4\]

(iii) By using an appropriate trigonometrical identity, deduce the exact value of

\[
\int_{0}^{\frac{\pi}{2}} \sin^2 x \, dx.
\] \[3\]

Q5.

3  Find the exact value of \( \int_{0}^{\frac{3\pi}{2}} (\cos 2x + \sin x) \, dx \). \[5\]

Q6.

3  The diagram shows the curve \( y = \frac{1}{1 + \sqrt{x}} \) for values of \( x \) from 0 to 2.

(i) Use the trapezium rule with two intervals to estimate the value of

\[
\int_{0}^{2} \frac{1}{1 + \sqrt{x}} \, dx,
\]

giving your answer correct to 2 decimal places. \[3\]

(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i). \[1\]

Q7.
Q8.

2. Show that \[ \int_{0}^{6} \frac{1}{x+2} \, dx = 2 \ln 2. \] \[ \text{[4]} \]

The diagram shows part of the curve \( y = xe^{-x} \). The shaded region \( R \) is bounded by the curve and by the lines \( x = 2 \), \( x = 3 \) and \( y = 0 \).

(i) Use the trapezium rule with two intervals to estimate the area of \( R \), giving your answer correct to 2 decimal places. \[ \text{[3]} \]

(ii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the area of \( R \). \[ \text{[1]} \]

Q9.

4. (a) Show that \( \int_{0}^{\pi} \cos 2x \, dx = \frac{1}{2} \). \[ \text{[2]} \]

(b) By using an appropriate trigonometrical identity, find the exact value of
\[ \int_{\frac{\pi}{4}}^{\pi} 3 \tan^2 x \, dx. \] \[ \text{[4]} \]

Q10.

6. (a) Find \( \int 4e^{(3 + e^{2x})} \, dx. \) \[ \text{[4]} \]

(b) Show that \( \int_{\frac{1}{2}}^{\pi} (3 + 2 \tan^2 \theta) \, d\theta = \frac{1}{2}(8 + \pi). \) \[ \text{[4]} \]

Q11.
The diagram shows the curve $y = \sqrt{1 + x^4}$. Region $A$ is bounded by the curve and the lines $x = 0$, $x = 2$ and $y = 0$. Region $B$ is bounded by the curve and the lines $x = 0$ and $y = 3$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of region $A$. Give your answer correct to 2 decimal places. [3]

(ii) Deduce an approximation to the area of region $B$ and explain why this approximation underestimates the true area of region $B$. [2]

Q12.

4 (a) Find the value of $\int_{0}^{\pi} \sin(\frac{1}{2}x) \, dx$. [3]

(b) Find $\int e^{-x}(1 + e^x) \, dx$. [3]

Q13.

7 (i) Show that $(2 \sin x + \cos x)^2$ can be written in the form $\frac{5}{2} + 2 \sin 2x - \frac{3}{2} \cos 2x$. [5]

(ii) Hence find the exact value of $\int_{0}^{\frac{\pi}{2}} (2 \sin x + \cos x)^2 \, dx$. [4]

Q14.
Q15.

3 (i) Show that $12 \sin^3 x \cos^2 x = \frac{3}{2} (1 - \cos 4x)$.

(ii) Hence show that

$$
\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 12 \sin^3 x \cos^2 x \, dx = \frac{\pi}{8} + \frac{3\sqrt{3}}{16}.
$$

Q16.

1 A curve is such that $\frac{dy}{dx} = \frac{4}{7 - 2x}$. The point (3, 2) lies on the curve. Find the equation of the curve.

Q17.
Q18. 

6 (a) Find the value of \(\int_{0}^{\frac{1}{2}\pi} (\sin 2x + \cos x) \, dx\). \([4]\)

(b) 

The diagram shows part of the curve \(y = \frac{1}{x + 1}\). The shaded region \(R\) is bounded by the curve and by the lines \(x = 1\), \(y = 0\) and \(x = p\).

(i) Find, in terms of \(p\), the area of \(R\). \([3]\)

(ii) Hence find, correct to 1 decimal place, the value of \(p\) for which the area of \(R\) is equal to 2. \([2]\)

Q19. 

7 (i) Given that \(y = \tan 2x\), find \(\frac{dy}{dx}\). \([2]\)

(ii) Hence, or otherwise, show that

\[\int_{0}^{\frac{1}{2}\pi} \sec^2 2x \, dx = \frac{1}{2}\sqrt{3},\]

and, by using an appropriate trigonometrical identity, find the exact value of \(\int_{0}^{\frac{1}{2}\pi} \tan^2 2x \, dx\). \([6]\)

(iii) Use the identity \(\cos 4x \equiv 2 \cos^2 2x - 1\) to find the exact value of

\[\int_{0}^{\frac{1}{2}\pi} \frac{1}{1 + \cos 4x} \, dx.\] \([2]\)

Q20. 

1 Show that

\[\int_{1}^{4} \frac{1}{2x + 1} \, dx = \frac{1}{2} \ln 3.\] \([4]\)
7 (i) Prove the identity

\[(\cos x + 3 \sin x)^2 = 5 - 4 \cos 2x + 3 \sin 2x\].

(ii) Using the identity, or otherwise, find the exact value of

\[\int_0^{\frac{\pi}{2}} (\cos x + 3 \sin x)^2 \, dx\].

Q21.

5 Show that \[\int_1^2 \left(\frac{1}{x} - \frac{4}{2x+1}\right) \, dx = \ln \frac{18}{25}\].

Q22.

5 (i) Express \(\cos^2 2x\) in terms of \(\cos 4x\).

(ii) Hence find the exact value of \[\int_0^{\frac{\pi}{4}} \cos^2 2x \, dx\].

Q23.

3 (i) Use the trapezium rule with two intervals to estimate the value of

\[\int_0^{\frac{\pi}{3}} \sec x \, dx\],

giving your answer correct to 2 decimal places.

(ii) Using a sketch of the graph of \(y = \sec x\) for \(0 \leq x \leq \frac{\pi}{3}\), explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i).

Q24.

8 (a) Find the exact value of \[\int_0^{\frac{\pi}{4}} (\sin 2x + \sec^2 x) \, dx\].

(b) Show that \[\int_1^4 \left(\frac{1}{2x} + \frac{1}{x+1}\right) \, dx = \ln 5\].

Q25.
Q26.

4  (a) Find \( \int e^{1-2x} \, dx \).  
    
(b) Express \( \sin^2 3x \) in terms of \( \cos 6x \) and hence find \( \int \sin^2 3x \, dx \).

Q27.

2  Show that \( \int_2^6 \frac{2}{4x+1} \, dx = \ln \frac{5}{3} \).

Q28.

8  (i) By first expanding \( \cos(2x + x) \), show that 
    
    \[ \cos 3x = 4 \cos^3 x - 3 \cos x. \]  
    
(ii) Hence show that 
    
    \[ \int_0^{\frac{\pi}{2}} (2 \cos^3 x - \cos x) \, dx = \frac{5}{12}. \]

Q29.

4  Find the exact value of the positive constant \( k \) for which 
    
    \[ \int_0^k e^{4x} \, dx = \int_0^{2k} e^x \, dx. \]

Q30.

4  (i) Express \( \cos^2 x \) in terms of \( \cos 2x \).
    
(ii) Hence show that 
    
    \[ \int_0^{\frac{\pi}{2}} (\cos^2 x + \sin 2x) \, dx = \frac{1}{8} \sqrt{3} + \frac{1}{12} \pi + \frac{1}{4}. \]

Q31.
Q32.

6 (a) Use the trapezium rule with two intervals to estimate the value of
\[ \int_0^1 \frac{1}{6 + 2e^x} \, dx, \]
giving your answer correct to 2 decimal places. [3]

(b) Find \[ \int \frac{(e^x - 2)^2}{e^{2x}} \, dx. \] [4]

Q33.

6 (a) Find \[ \int 4e^{-3x} \, dx. \] [2]

(b) Show that \[ \int_1^3 \frac{6}{3x - 1} \, dx = \ln 16. \] [5]

Q34.

1 (i) Find \[ \int \frac{2}{4x - 1} \, dx. \] [2]

(ii) Hence find \[ \int_1^7 \frac{2}{4x - 1} \, dx, \] expressing your answer in the form \( \ln \sigma \), where \( \sigma \) is an integer. [3]

Q35.
6 (a) Find \( \int (\sin x - \cos x)^2 \, dx \). \[4\]

(b) (i) Use the trapezium rule with 2 intervals to estimate the value of
\[\int_{\frac{1}{2}\pi}^{\frac{1}{2}\pi} \csc x \, dx,\]
giving your answer correct to 3 decimal places. \[3\]

(ii) Using a sketch of the graph of \( y = \csc x \) for \( 0 < x \leq \frac{1}{2}\pi \), explain whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part (i). \[2\]

Q36.

5 (i) Prove that \( \tan \theta + \cot \theta = \frac{2}{\sin 2\theta} \). \[3\]

(ii) Hence

(a) find the exact value of \( \tan \frac{1}{2}\pi + \cot \frac{1}{2}\pi \), \[2\]

(b) evaluate \( \int_{0}^{\frac{1}{2}\pi} \frac{6}{\tan \theta + \cot \theta} \, d\theta \). \[3\]

Q37.

6 (a) Show that \( \int_{6}^{16} \frac{6}{2x - 7} \, dx = \ln 125 \). \[5\]

(b) Use the trapezium rule with four intervals to find an approximation to
\[\int_{1}^{17} \log_{10} x \, dx,\]
giving your answer correct to 3 significant figures. \[3\]

Q38.

3 (a) Find \( \int 4 \cos \left( \frac{1}{3}x + 2 \right) \, dx \). \[2\]

(b) Use the trapezium rule with three intervals to find an approximation to
\[\int_{0}^{12} \sqrt{(4 + x^2)} \, dx,\]
giving your answer correct to 3 significant figures. \[3\]

Q39.
1 Use the trapezium rule with four intervals to find an approximation to
\[ \int_1^5 |2^x - 8| \, dx. \] [3]

Q40.

3 (a) Find \( \int 4 \cos^2 \left( \frac{1}{2} \theta \right) \, d\theta \). [3]

(b) Find the exact value of \( \int_{-1}^{6} \frac{1}{2x+3} \, dx. \) [4]

Q41.

2 (i) Find \( \int_0^a (e^{-x} + 6e^{-3x}) \, dx \), where \( a \) is a positive constant. [4]

(ii) Deduce the value of \( \int_0^\infty (e^{-x} + 6e^{-3x}) \, dx \). [1]

P3 (variant 1 and 3)

Q1.

4 (i) Using the expansions of \( \cos(3x-x) \) and \( \cos(3x+x) \), prove that
\[ \frac{1}{2} (\cos 2x - \cos 4x) = \sin 3x \sin x. \] [3]

(ii) Hence show that
\[ \int_{\frac{\pi}{4}}^{\pi} \sin 3x \sin x \, dx = \frac{\pi}{8\sqrt{3}}. \] [3]

Q2.

8 (i) Express \( \frac{2}{(x+1)(x+3)} \) in partial fractions. [2]

(ii) Using your answer to part (i), show that
\[ \left( \frac{2}{(x+1)(x+3)} \right)^2 = \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}. \] [2]

(iii) Hence show that \( \int_0^1 \frac{4}{(x+1)^2(x+3)^2} \, dx = \frac{7}{13} - \ln \frac{3}{2} \). [5]

Q3.
7. (i) Prove the identity \( \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta \). [4]

(ii) Using this result, find the exact value of

\[
\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^3 \theta \, d\theta.
\] [4]

Q4.

7. The integral \( I \) is defined by \( I = \int_0^2 4t^3 \ln(t^2 + 1) \, dt \).

(i) Use the substitution \( x = t^2 + 1 \) to show that \( I = \int_1^5 (2x - 2) \ln x \, dx \). [3]

(ii) Hence find the exact value of \( I \). [5]

Q5.

10. The number of birds of a certain species in a forested region is recorded over several years. At time \( t \) years, the number of birds is \( N \), where \( N \) is treated as a continuous variable. The variation in the number of birds is modelled by

\[
\frac{dN}{dt} = \frac{N(1800 - N)}{3600}.
\]

It is given that \( N = 300 \) when \( t = 0 \).

(i) Find an expression for \( N \) in terms of \( t \). [9]

(ii) According to the model, how many birds will there be after a long time? [1]

Q6.

3. Show that \( \int_0^1 (1 - x)e^{\frac{x}{2}} \, dx = 4e^{\frac{1}{2}} - 2 \). [5]

Q7.
In a chemical reaction, a compound \( X \) is formed from two compounds \( Y \) and \( Z \). The masses in grams of \( X \), \( Y \) and \( Z \) present at time \( t \) seconds after the start of the reaction are \( x \), \( 10 - x \) and \( 20 - x \) respectively. At any time the rate of formation of \( X \) is proportional to the product of the masses of \( Y \) and \( Z \) present at the time. When \( t = 0 \), \( x = 0 \) and \( \frac{dx}{dt} = 2 \).

(i) Show that \( x \) and \( t \) satisfy the differential equation

\[
\frac{dx}{dt} = 0.01(10-x)(20-x).
\]

(ii) Solve this differential equation and obtain an expression for \( x \) in terms of \( t \).

(iii) State what happens to the value of \( x \) when \( t \) becomes large.

Q8.

The variables \( x \) and \( y \) are related by the differential equation

\[
\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.
\]

It is given that \( y = 2 \) when \( x = 0 \). Solve the differential equation and hence find the value of \( y \) when \( x = 0.5 \), giving your answer correct to 2 decimal places.

Q9.

Given that \( y = 0 \) when \( x = 1 \), solve the differential equation

\[
x^3 \frac{dy}{dx} = y^2 + 4,
\]

obtaining an expression for \( y^2 \) in terms of \( x \).

Q10.

Given that \( x = 1 \) when \( t = 0 \), solve the differential equation

\[
\frac{dx}{dt} = \frac{1 - x}{x - 4},
\]

obtaining an expression for \( x^2 \) in terms of \( t \).

Q11.

By first expressing \( \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2} \) in partial fractions, show that

\[
\int_0^4 \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2} \, dx = 8 - \ln 9.
\]

[10]
Q12.

In a certain chemical process a substance $A$ reacts with another substance $B$. The masses in grams of $A$ and $B$ present at time $t$ seconds after the start of the process are $x$ and $y$ respectively. It is given that \( \frac{dy}{dt} = -0.6xy \) and $x = 5e^{-3t}$. When $t = 0$, $y = 70$.

(i) Form a differential equation in $y$ and $t$. Solve this differential equation and obtain an expression for $y$ in terms of $t$. [6]

(ii) The percentage of the initial mass of $B$ remaining at time $t$ is denoted by $p$. Find the exact value approached by $p$ as $t$ becomes large. [2]

Q13.

Let $f(x) = \frac{4x^2 - 7x - 1}{(x + 1)(2x - 3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_{2}^{6} f(x) \, dx = 8 - \ln\left(\frac{49}{3}\right)$. [5]

Q14.

(a) Show that $\int_{2}^{4} 4x \ln x \, dx = 56 \ln 2 - 12$. [5]

(b) Use the substitution $\mu = \sin 4x$ to find the exact value of $\int_{0}^{\frac{\pi}{4}} \cos^3 4x \, dx$. [5]

Q15.
10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, \( t \) minutes later, the volume of liquid in the tank is \( V \) cm\(^3\). The liquid is flowing into the tank at a constant rate of 80 cm\(^3\) per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to \( kV \) cm\(^3\) per minute where \( k \) is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

\[
V = \frac{1}{k} (80 - 80e^{-kt}).
\]  

(ii) It is observed that \( V = 500 \) when \( t = 15 \), so that \( k \) satisfies the equation

\[
k = \frac{4 - 4e^{-15k}}{25}.
\]

Use an iterative formula, based on this equation, to find the value of \( k \) correct to 2 significant figures. Use an initial value of \( k = 0.1 \) and show the result of each iteration to 4 significant figures.  

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time.

4 (i) Express \( \sqrt{3} \cos x + \sin x \) in the form \( R \cos (x - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{\pi}{2} \), giving the exact values of \( R \) and \( \alpha \).  

(ii) Hence show that

\[
\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{((\sqrt{3}) \cos x + \sin x)^2} \, dx = \frac{1}{4\sqrt{3}}.
\]

8 The variables \( x \) and \( t \) satisfy the differential equation

\[
\frac{dx}{dt} = \frac{k - x^3}{2x^2},
\]

for \( t > 0 \), where \( k \) is a constant. When \( t = 1, x = 1 \) and when \( t = 4, x = 2 \).

(i) Solve the differential equation, finding the value of \( k \) and obtaining an expression for \( x \) in terms of \( t \).  

(ii) State what happens to the value of \( x \) as \( t \) becomes large.

Q16.

Q17.

Q18.
Q19.

10 In a model of the expansion of a sphere of radius \( r \) cm, it is assumed that, at time \( t \) seconds after the start, the rate of increase of the surface area of the sphere is proportional to its volume. When \( t = 0 \), \( \frac{dr}{dt} = 2 \).

(i) Show that \( r \) satisfies the differential equation
\[
\frac{dr}{dt} = 0.08r^2.
\]

[The surface area \( A \) and volume \( V \) of a sphere of radius \( r \) are given by the formulae \( A = 4\pi r^2 \), \( V = \frac{4}{3}\pi r^3 \).]

(ii) Solve this differential equation, obtaining an expression for \( r \) in terms of \( t \).

(iii) Deduce from your answer to part (ii) the set of values that \( t \) can take, according to this model.

Q20.

5 Let \( I = \int_{0}^{1} \frac{x^2}{\sqrt{(4-x^2)}} \, dx \).

(i) Using the substitution \( x = 2\sin \theta \), show that
\[
I = \int_{0}^{\frac{\pi}{2}} 4\sin^2 \theta \, d\theta.
\]

(ii) Hence find the exact value of \( I \).

Q21.
A certain substance is formed in a chemical reaction. The mass of substance formed \( t \) seconds after the start of the reaction is \( x \) grams. At any time the rate of formation of the substance is proportional to \( 20 - x \). When \( t = 0, x = 0 \) and \( \frac{dx}{dt} = 1 \).

(i) Show that \( x \) and \( t \) satisfy the differential equation

\[
\frac{dx}{dt} = 0.05(20 - x).
\]

(ii) Find, in any form, the solution of this differential equation.

(iii) Find \( x \) when \( t = 10 \), giving your answer correct to 1 decimal place.

(iv) State what happens to the value of \( x \) as \( t \) becomes very large.

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Show that \( \int_{0}^{7} \frac{2x + 7}{(2x + 1)(x + 2)} \, dx = \ln 50 \). [7]

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A biologist is investigating the spread of a weed in a particular region. At time \( t \) weeks after the start of the investigation, the area covered by the weed is \( A \) m\(^2\). The biologist claims that the rate of increase of \( A \) is proportional to \( \sqrt{2A - 5} \).

(i) Write down a differential equation representing the biologist’s claim. [1]

(ii) At the start of the investigation, the area covered by the weed was 7 m\(^2\) and, 10 weeks later, the area covered was 27 m\(^2\). Assuming that the biologist’s claim is correct, find the area covered 20 weeks after the start of the investigation. [9]

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The variables \( x \) and \( \theta \) are related by the differential equation

\[
\sin 2\theta \frac{dx}{d\theta} = (x + 1) \cos 2\theta,
\]

where \( 0 < \theta < \frac{1}{2} \pi \). When \( \theta = \frac{1}{4} \pi, x = 0 \). Solve the differential equation, obtaining an expression for \( x \) in terms of \( \theta \), and simplifying your answer as far as possible. [7]
Q26.

4. During an experiment, the number of organisms present at time $t$ days is denoted by $N$, where $N$ is treated as a continuous variable. It is given that

$$\frac{dN}{dt} = 1.2e^{-0.02t}N^{0.5}.$$ 

When $t = 0$, the number of organisms present is 100.

(i) Find an expression for $N$ in terms of $t$.  

(ii) State what happens to the number of organisms present after a long time.

Q27.

10. (i) Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

$$\int_0^{\frac{1}{n}} (\tan^{n+2} x + \tan^n x) \, dx = \frac{1}{n+1}. \quad [4]$$

(ii) Hence find the exact value of

(a) $\int_0^{\frac{1}{2}} (\sec^3 x - \sec^3 x) \, dx$,  

(b) $\int_0^{\frac{1}{2}} (\tan^3 x + 5 \tan^7 x + 5 \tan^5 x + \tan^3 x) \, dx$.  

Q28.

6. The variables $x$ and $y$ are related by the differential equation

$$x \frac{dy}{dx} = 1 - y^2.$$ 

When $x = 2, y = 0$. Solve the differential equation, obtaining an expression for $y$ in terms of $x$.  

Q29.
4 The variables \( x \) and \( y \) are related by the differential equation

\[
(x^2 + 4) \frac{dy}{dx} = 6xy.
\]

It is given that \( y = 32 \) when \( x = 0 \). Find an expression for \( y \) in terms of \( x \). \[6\]

Q30.

7

The diagram shows part of the curve \( y = \sin^3 2x \cos^3 2x \). The shaded region shown is bounded by the curve and the \( x \)-axis and its exact area is denoted by \( A \).

(i) Use the substitution \( u = \sin 2x \) in a suitable integral to find the value of \( A \). \[6\]

(ii) Given that \( \int_{0}^{\pi} |\sin^3 2x \cos^3 2x| \, dx = 40A \), find the value of the constant \( k \). \[2\]

Q31.

3 Find the exact value of \( \int_{1}^{4} \frac{\ln x}{\sqrt{x}} \, dx \). \[5\]

Q32.

5 (i) Prove that \( \cot \theta + \tan \theta = 2 \csc 2\theta \). \[3\]

(ii) Hence show that \( \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \csc 2\theta \, d\theta = \frac{1}{2} \ln 3 \). \[4\]

Q33.
A tank containing water is in the form of a cone with vertex $C$. The axis is vertical and the semi-vertical angle is $60^\circ$, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is $H$. At this instant, a tap at $C$ is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where $h$ is the depth of water at time $t$. The tank becomes empty when $t = 60$.

(i) Show that $h$ and $t$ satisfy a differential equation of the form
\[
\frac{dh}{dt} = -Ah^{-\frac{3}{2}},
\]
where $A$ is a positive constant. \[4\]

(ii) Solve the differential equation given in part (i) and obtain an expression for $t$ in terms of $h$ and $H$. \[6\]

(iii) Find the time at which the depth reaches $\frac{1}{2}H$. \[1\]

[The volume $V$ of a cone of vertical height $h$ and base radius $r$ is given by $V = \frac{1}{3}\pi r^2 h$.]

Q34.

2 Use the substitution $u = 3x + 1$ to find \[\int \frac{3x}{3x + 1} \, dx.\] \[4\]

Q35.
A particular solution of the differential equation
\[ 3y^2 \frac{dy}{dx} = 4(y^3 + 1) \cos^2 x \]
is such that \( y = 2 \) when \( x = 0 \). The diagram shows a sketch of the graph of this solution for \( 0 \leq x \leq 2\pi \); the graph has stationary points at \( A \) and \( B \). Find the \( y \)-coordinates of \( A \) and \( B \), giving each coordinate correct to 1 decimal place. [10]

Q36.

2. Use the substitution \( u = 1 + 3 \tan x \) to find the exact value of
\[ \int_0^{\frac{\pi}{4}} \frac{\sqrt{(1 + 3 \tan x)}}{\cos^2 x} \, dx. \] [5]

Q37.

4. The variables \( x \) and \( y \) are related by the differential equation
\[ \frac{dy}{dx} = \frac{6y e^{3x}}{2 + e^{3x}}. \]
Given that \( y = 36 \) when \( x = 0 \), find an expression for \( y \) in terms of \( x \). [6]

Q38.

5. The variables \( x \) and \( \theta \) satisfy the differential equation
\[ 2 \cos^2 \theta \frac{dx}{d\theta} = \sqrt{(2x + 1)}, \]
and \( x = 0 \) when \( \theta = \frac{1}{4} \pi \). Solve the differential equation and obtain an expression for \( x \) in terms of \( \theta \). [7]
Q39.

8. Let \( f(x) = \frac{6 + 6x}{(2 - x)(2 + x^2)} \).
   
   (i) Express \( f(x) \) in the form \( \frac{A}{2-x} + \frac{Bx + C}{2 + x^2} \). \[4\]
   
   (ii) Show that \( \int_{-1}^{1} f(x) \, dx = 3 \ln 3 \). \[5\]

Q40.

2. (i) Use the trapezium rule with 3 intervals to estimate the value of

\[ \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \csc x \, dx, \]


giving your answer correct to 2 decimal places. \[3\]

(ii) Using a sketch of the graph of \( y = \csc x \), explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part (i). \[2\]

Q41.

7. In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is \( R \) million dollars when the rate of tax is \( x \) dollars per litre. The variation of \( R \) with \( x \) is modelled by the differential equation

\[ \frac{dR}{dx} = R \left( \frac{1}{x} - 0.57 \right), \]

where \( R \) and \( x \) are taken to be continuous variables. When \( x = 0.5, R = 16.8 \).

(i) Solve the differential equation and obtain an expression for \( R \) in terms of \( x \). \[6\]

(ii) This model predicts that \( R \) cannot exceed a certain amount. Find this maximum value of \( R \). \[3\]

Q42.

6. It is given that \( I = \int_{0}^{0.3} (1 + 3x^2)^{-2} \, dx \).

(i) Use the trapezium rule with 3 intervals to find an approximation to \( I \), giving the answer correct to 3 decimal places. \[3\]

(ii) For small values of \( x \), \( (1 + 3x^2)^{-2} \approx 1 + ax^2 + bx^4 \). Find the values of the constants \( a \) and \( b \).

Hence, by evaluating \( \int_{0}^{0.3} (1 + ax^2 + bx^4) \, dx \), find a second approximation to \( I \), giving the answer correct to 3 decimal places. \[5\]
Q43.

8 The variables $x$ and $y$ are related by the differential equation

$$\frac{dy}{dx} = \frac{1}{3} xy^{\frac{1}{2}} \sin \left( \frac{1}{3} x \right).$$

(i) Find the general solution, giving $y$ in terms of $x$. \hspace{1cm} [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of $y$ when $x = 25$. \hspace{1cm} [3]

Q44.

10 By first using the substitution $u = e^x$, show that

$$\int_{0}^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2} \, dx = \ln \left( \frac{8}{3} \right).$$ \hspace{1cm} [10]