These are P2 questions (all variants) as the syllabus is same as P3 :

Q1.

2. The polynomial \( x^4 - 9x^2 - 6x - 1 \) is denoted by \( f(x) \).
   
   (i) Find the value of the constant \( a \) for which  
       \[ f(x) = (x^2 + ax + 1)(x^2 - ax - 1). \]  
       \[ \text{[3]} \] 
   
   (ii) Hence solve the equation \( f(x) = 0 \), giving your answers in an exact form.  
        \[ \text{[3]} \]

Q2.

3. The cubic polynomial \( 2x^3 + ax^2 - 13x - 6 \) is denoted by \( f(x) \). It is given that \((x - 3)\) is a factor of \( f(x) \).
   
   (i) Find the value of \( a \).  
       \[ \text{[2]} \] 
   
   (ii) When \( a \) has this value, solve the equation \( f(x) = 0 \).  
        \[ \text{[4]} \]

Q3.

4. The polynomial \( x^3 - x^2 + ax + b \) is denoted by \( p(x) \). It is given that \((x + 1)\) is a factor of \( p(x) \) and that when \( p(x) \) is divided by \((x - 2)\) the remainder is 12.
   
   (i) Find the values of \( a \) and \( b \).  
       \[ \text{[5]} \] 
   
   (ii) When \( a \) and \( b \) have these values, factorise \( p(x) \).  
        \[ \text{[2]} \]

Q4.

4. The cubic polynomial \( ax^3 + bx^2 - 3x - 2 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \((x - 1)\) and \((x + 2)\) are factors of \( p(x) \).
   
   (i) Find the values of \( a \) and \( b \).  
       \[ \text{[5]} \] 
   
   (ii) When \( a \) and \( b \) have these values, find the other linear factor of \( p(x) \).  
        \[ \text{[2]} \]

Q5.

4. The polynomial \( 2x^3 - 3x^2 + ax + b \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \((x - 2)\) is a factor of \( p(x) \), and that when \( p(x) \) is divided by \((x + 2)\) the remainder is \(-20\).
   
   (i) Find the values of \( a \) and \( b \).  
       \[ \text{[5]} \] 
   
   (ii) When \( a \) and \( b \) have these values, find the remainder when \( p(x) \) is divided by \((x^2 - 4)\).  
        \[ \text{[3]} \]
Q6.

4 The polynomial \(2x^3 + 7x^2 + ax + b\), where \(a\) and \(b\) are constants, is denoted by \(p(x)\). It is given that \((x + 1)\) is a factor of \(p(x)\), and that when \(p(x)\) is divided by \((x + 2)\) the remainder is 5. Find the values of \(a\) and \(b\). [5]

Q7.

6 The polynomial \(x^3 + ax^2 + bx + 6\), where \(a\) and \(b\) are constants, is denoted by \(p(x)\). It is given that \((x - 2)\) is a factor of \(p(x)\), and that when \(p(x)\) is divided by \((x - 1)\) the remainder is 4.

(i) Find the values of \(a\) and \(b\). [5]

(ii) When \(a\) and \(b\) have these values, find the other two linear factors of \(p(x)\). [3]

Q8.

4 The polynomial \(x^3 + 3x^2 + 4x + 2\) is denoted by \(f(x)\).

(i) Find the quotient and remainder when \(f(x)\) is divided by \(x^2 + x - 1\). [4]

(ii) Use the factor theorem to show that \((x + 1)\) is a factor of \(f(x)\). [2]

Q9.

7 The polynomial \(2x^3 + ax^2 + bx + 6\), where \(a\) and \(b\) are constants, is denoted by \(p(x)\). It is given that when \(p(x)\) is divided by \((x - 3)\) the remainder is 30, and that when \(p(x)\) is divided by \((x + 1)\) the remainder is 18.

(i) Find the values of \(a\) and \(b\). [5]

(ii) When \(a\) and \(b\) have these values, verify that \((x - 2)\) is a factor of \(p(x)\) and hence factorise \(p(x)\) completely. [4]

Q10.

4 The polynomial \(f(x)\) is defined by

\[f(x) = 3x^3 + ax^2 + ax + a,\]

where \(a\) is a constant.

(i) Given that \((x + 2)\) is a factor of \(f(x)\), find the value of \(a\). [2]

(ii) When \(a\) has the value found in part (i), find the quotient when \(f(x)\) is divided by \((x + 2)\). [3]

Q11.
Q12.

The cubic polynomial \( p(x) \) is defined by
\[
p(x) = 6x^3 + ax^2 + bx + 10,
\]
where \( a \) and \( b \) are constants. It is given that \( (x + 2) \) is a factor of \( p(x) \) and that, when \( p(x) \) is divided by \( (x + 1) \), the remainder is 24.

(i) Find the values of \( a \) and \( b \). [5]

(ii) When \( a \) and \( b \) have these values, factorise \( p(x) \) completely. [3]

Q13.

The polynomial \( p(x) \) is defined by
\[
p(x) = ax^3 - 3x^2 - 5x + a + 4,
\]
where \( a \) is a constant.

(i) Given that \( (x - 2) \) is a factor of \( p(x) \), find the value of \( a \). [2]

(ii) When \( a \) has this value,
   (a) factorise \( p(x) \) completely. [3]
   (b) find the remainder when \( p(x) \) is divided by \( (x + 1) \). [2]

Q14.

The polynomial \( ax^3 - 5x^2 + bx + 9 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (2x + 3) \) is a factor of \( p(x) \), and that when \( p(x) \) is divided by \( (x + 1) \) the remainder is 8.

(i) Find the values of \( a \) and \( b \). [5]

(ii) When \( a \) and \( b \) have these values, factorise \( p(x) \) completely. [3]

Q15.
3 (i) The polynomial $2x^3 + ax^2 - ax - 12$, where $a$ is a constant, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$. Find the value of $a$. [2]

(ii) When $a$ has this value, find the remainder when $p(x)$ is divided by $(x + 3)$. [2]

Q16.

2 The cubic polynomial $2x^3 + ax^2 + b$ is denoted by $f(x)$. It is given that $(x + 1)$ is a factor of $f(x)$, and that when $f(x)$ is divided by $(x + 2)$ the remainder is $-5$. Find the values of $a$ and $b$. [5]

Q17.

3 The polynomial $x^4 - 6x^2 + x + a$ is denoted by $f(x)$.

(i) It is given that $(x + 1)$ is a factor of $f(x)$. Find the value of $a$. [2]

(ii) When $a$ has this value, verify that $(x - 2)$ is also a factor of $f(x)$ and hence factorise $f(x)$ completely. [4]

Q18.

4 The cubic polynomial $2x^3 - 5x^2 + ax + b$ is denoted by $f(x)$. It is given that $(x - 2)$ is a factor of $f(x)$, and that when $f(x)$ is divided by $(x + 1)$ the remainder is $-6$. Find the values of $a$ and $b$. [5]

Q19.

2 The polynomial $x^3 + 2x^2 + 2x + 3$ is denoted by $p(x)$.

(i) Find the remainder when $p(x)$ is divided by $x - 1$. [2]

(ii) Find the quotient and remainder when $p(x)$ is divided by $x^2 + x - 1$. [4]

Q20.

3 The polynomial $4x^3 - 7x + a$, where $a$ is a constant, is denoted by $p(x)$. It is given that $(2x - 3)$ is a factor of $p(x)$.

(i) Show that $a = -3$. [2]

(ii) Hence, or otherwise, solve the equation $p(x) = 0$. [4]

Q21.
Q22.

2 The polynomial \(2x^3 - x^2 + ax - 6\), where \(a\) is a constant, is denoted by \(p(x)\). It is given that \((x + 2)\) is a factor of \(p(x)\).

(i) Find the value of \(a\). \(\quad [2]\)

(ii) When \(a\) has this value, factorise \(p(x)\) completely. \(\quad [3]\)

Q23.

3 The polynomial \(4x^3 - 8x^2 + ax - 3\), where \(a\) is a constant, is denoted by \(p(x)\). It is given that \((2x + 1)\) is a factor of \(p(x)\).

(i) Find the value of \(a\). \(\quad [2]\)

(ii) When \(a\) has this value, factorise \(p(x)\) completely. \(\quad [4]\)

Q24.

5 The polynomial \(ax^3 + bx^2 - 5x + 2\), where \(a\) and \(b\) are constants, is denoted by \(p(x)\). It is given that \((x + 1)\) and \((x - 2)\) are factors of \(p(x)\).

(i) Find the values of \(a\) and \(b\). \(\quad [5]\)

(ii) When \(a\) and \(b\) have these values, find the other linear factor of \(p(x)\). \(\quad [2]\)

Q25.

7 The polynomial \(3x^3 + 2x^2 + ax + b\), where \(a\) and \(b\) are constants, is denoted by \(p(x)\). It is given that \((x - 1)\) is a factor of \(p(x)\), and that when \(p(x)\) is divided by \((x - 2)\) the remainder is 10.

(i) Find the values of \(a\) and \(b\). \(\quad [5]\)

(ii) When \(a\) and \(b\) have these values, solve the equation \(p(x) = 0\). \(\quad [4]\)

Q26.
Q27.

3 The polynomial \( x^3 + 4x^2 + ax + 2 \), where \( a \) is a constant, is denoted by \( p(x) \). It is given that the remainder when \( p(x) \) is divided by \((x + 1)\) is equal to the remainder when \( p(x) \) is divided by \((x - 2)\).

(i) Find the value of \( a \). \[3\]

(ii) When \( a \) has this value, show that \((x - 1)\) is a factor of \( p(x) \) and find the quotient when \( p(x) \) is divided by \((x - 1)\). \[3\]

Q28.

5 The polynomial \( 4x^3 + ax^2 + 9x + 9 \), where \( a \) is a constant, is denoted by \( p(x) \). It is given that when \( p(x) \) is divided by \((2x - 1)\) the remainder is 10.

(i) Find the value of \( a \) and hence verify that \((x - 3)\) is a factor of \( p(x) \). \[3\]

(ii) When \( a \) has this value, solve the equation \( p(x) = 0 \). \[4\]

Q29.

7 The polynomial \( ax^3 - 3x^2 - 11x + b \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \((x + 2)\) is a factor of \( p(x) \), and that when \( p(x) \) is divided by \((x + 1)\) the remainder is 12.

(i) Find the values of \( a \) and \( b \). \[5\]

(ii) When \( a \) and \( b \) have these values, factorise \( p(x) \) completely. \[3\]

Q30.

6 (i) The polynomial \( x^4 + ax^3 - x^2 + bx + 2 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \((x - 1)\) and \((x + 2)\) are factors of \( p(x) \). Find the values of \( a \) and \( b \). \[5\]

(ii) When \( a \) and \( b \) have these values, find the quotient when \( p(x) \) is divided by \( x^2 + x - 2 \). \[3\]

Q31.
Q32.

4  
(i) The polynomial \( x^3 + ax^2 + bx + 8 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that when \( p(x) \) is divided by \( x - 3 \) the remainder is 14, and that when \( p(x) \) is divided by \( x + 2 \) the remainder is 24. Find the values of \( a \) and \( b \). [5]  
(ii) When \( a \) and \( b \) have these values, find the quotient when \( p(x) \) is divided by \( x^2 + 2x - 8 \) and hence solve the equation \( p(x) = 0 \). [4]

Q33.

4  
(i) The polynomial \( ax^3 + bx^2 - 25x - 6 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (x - 3) \) and \( (x + 2) \) are factors of \( p(x) \). Find the values of \( a \) and \( b \). [5]  
(ii) When \( a \) and \( b \) have these values, factorise \( p(x) \) completely. [2]

Q34.

3  
(i) Find the quotient when \( 6x^4 - x^3 - 26x^2 + 4x + 15 \) is divided by \( (x^2 - 4) \), and confirm that the remainder is 7. [3]  
(ii) Hence solve the equation \( 6x^4 - x^3 - 26x^2 + 4x + 8 = 0 \). [3]

Q35.

6  
The polynomial \( p(x) \) is defined by

\[
p(x) = x^3 + 2x + a,
\]

where \( a \) is a constant.  
(i) Given that \( (x + 2) \) is a factor of \( p(x) \), find the value of \( a \). [2]  
(ii) When \( a \) has this value, find the quotient when \( p(x) \) is divided by \( (x + 2) \) and hence show that the equation \( p(x) = 0 \) has exactly one real root. [5]

Q36.
P3 (variant1 and 3)

Q1.

4 The polynomial \( f(x) \) is defined by
\[
f(x) = 12x^3 + 25x^2 - 4x - 12.
\]

(i) Show that \( f(-2) = 0 \) and factorise \( f(x) \) completely. [4]

(ii) Given that
\[
12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,
\]
state the value of \( 3^y \) and hence find \( y \) correct to 3 significant figures. [3]

Q2.

5 The polynomial \( ax^3 + bx^2 + 5x - 2 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (2x - 1) \) is a factor of \( p(x) \) and that when \( p(x) \) is divided by \( (x - 2) \) the remainder is 12.

(i) Find the values of \( a \) and \( b \). [5]

(ii) When \( a \) and \( b \) have these values, find the quadratic factor of \( p(x) \). [2]

Q3.
3 The polynomial $p(x)$ is defined by

$$p(x) = x^3 - 3ax + 4a,$$

where $a$ is a constant.

(i) Given that $(x - 2)$ is a factor of $p(x)$, find the value of $a$. [2]

(ii) When $a$ has this value,

(a) factorise $p(x)$ completely, [3]

(b) find all the roots of the equation $p(x^2) = 0$. [2]

Q4.

1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

Q5.

5 The polynomial $8x^3 + ax^2 + bx + 3$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of $a$ and $b$. [5]

(ii) When $a$ and $b$ have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

Q6.

10 The polynomial $p(z)$ is defined by

$$p(z) = z^3 + mz^2 + 24z + 32,$$

where $m$ is a constant. It is given that $(z + 2)$ is a factor of $p(z)$.

(i) Find the value of $m$. [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $p(z) = 0$, [5]

(b) the six roots of the equation $p(z^2) = 0$. [6]

Q7.

3 The polynomial $x^3 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of $a$. [4]

(ii) When $a$ has this value, find the real roots of the equation $p(x) = 0$. [2]
Q8.

7 The polynomial \( p(x) \) is defined by
\[
p(x) = ax^3 - x^2 + 4x - a,
\]
where \( a \) is a constant. It is given that \( (2x - 1) \) is a factor of \( p(x) \).

(i) Find the value of \( a \) and hence factorise \( p(x) \). [4]

(ii) When \( a \) has the value found in part (i), express \( \frac{8x - 13}{p(x)} \) in partial fractions. [5]

Q9.

3 The polynomial \( f(x) \) is defined by
\[
f(x) = x^3 + ax^2 - ax + 14,
\]
where \( a \) is a constant. It is given that \( (x + 2) \) is a factor of \( f(x) \).

(i) Find the value of \( a \). [2]

(ii) Show that, when \( a \) has this value, the equation \( f(x) = 0 \) has only one real root. [3]

Q10.

3 The polynomial \( ax^3 + bx^2 + x + 3 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (3x + 1) \) is a factor of \( p(x) \), and that when \( p(x) \) is divided by \( (x - 2) \) the remainder is 21. Find the values of \( a \) and \( b \). [5]

Q11.

3 The polynomial \( 4x^3 + ax^2 + bx - 2 \), where \( a \) and \( b \) are constants, is denoted by \( p(x) \). It is given that \( (x + 1) \) and \( (x + 2) \) are factors of \( p(x) \).

(i) Find the values of \( a \) and \( b \). [4]

(ii) When \( a \) and \( b \) have these values, find the remainder when \( p(x) \) is divided by \( (x^2 + 1) \). [3]