These are P2 questions (all variants) as the syllabus is same as P3 :)

Q1.

4 (i) Show that the equation
\[ \tan(45^\circ + x) = 4 \tan(45^\circ - x) \]
can be written in the form
\[ 3 \tan^2 x - 10 \tan x + 3 = 0. \] [4]

(ii) Hence solve the equation
\[ \tan(45^\circ + x) = 4 \tan(45^\circ - x), \]
for \(0^\circ < x < 90^\circ\). [3]

Q2.

4 (i) Express \(3 \sin \theta + 4 \cos \theta\) in the form \(R \sin(\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), giving the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence solve the equation
\[ 3 \sin \theta + 4 \cos \theta = 4.5, \]
giving all solutions in the interval \(0^\circ \leq \theta \leq 360^\circ\), correct to 1 decimal place. [4]

(iii) Write down the least value of \(3 \sin \theta + 4 \cos \theta + 7\) as \(\theta\) varies. [1]

Q3.

2 (i) Prove the identity
\[ \cos(x + 30^\circ) + \sin(x + 60^\circ) \equiv (\sqrt{3}) \cos x. \] [3]

(ii) Hence solve the equation
\[ \cos(x + 30^\circ) + \sin(x + 60^\circ) = 1, \]
for \(0^\circ < x < 90^\circ\). [2]

Q4.

5 (i) Express \(5 \cos \theta - \sin \theta\) in the form \(R \cos(\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), giving the exact value of \(R\) and the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence solve the equation
\[ 5 \cos \theta - \sin \theta = 4, \]
giving all solutions in the interval \(0^\circ \leq \theta \leq 360^\circ\). [4]
Q5.
5 Solve the equation $\sec x = 4 - 2 \tan^2 x$, giving all solutions in the interval $0^\circ \leq x \leq 180^\circ$. [6]

Q6.
3 (i) Show that the equation $\tan(x + 45^\circ) = 6 \tan x$ can be written in the form
$$6 \tan^2 x - 5 \tan x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\tan(x + 45^\circ) = 6 \tan x$, for $0^\circ < x < 180^\circ$. [3]

Q7.
8 (i) Express $4 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places. [3]

(ii) Solve the equation $4 \sin \theta - 6 \cos \theta = 3$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

(iii) Find the greatest and least possible values of $(4 \sin \theta - 6 \cos \theta)^2$ as $\theta$ varies. [2]

Q8.
8 (i) Express $4 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places. [3]

(ii) Solve the equation $4 \sin \theta - 6 \cos \theta = 3$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

(iii) Find the greatest and least possible values of $(4 \sin \theta - 6 \cos \theta)^2$ as $\theta$ varies. [2]

Q9.
8 (i) Prove that $\sin^2 2\theta(\cosec^2 \theta - \sec^2 \theta) = 4 \cos 2\theta$. [3]

(ii) Hence
(a) solve for $0^\circ < \theta < 180^\circ$ the equation $\sin^2 2\theta(\cosec^2 \theta - \sec^2 \theta) = 3$, [4]
(b) find the exact value of $\cosec^2 15^\circ - \sec^2 15^\circ$. [2]

Q10.
4. (i) Given that \(35 + \sec^2 \theta = 12 \tan \theta\), find the value of \(\tan \theta\). [3]

(ii) Hence, showing the use of an appropriate formula in each case, find the exact value of
(a) \(\tan(\theta - 45^\circ)\), [2]
(b) \(\tan 2\theta\). [2]

Q11.

4 (i) Express \(9 \sin \theta - 12 \cos \theta\) in the form \(R \sin(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). Give the value of \(\alpha\) correct to 2 decimal places. [3]

Hence

(ii) solve the equation \(9 \sin \theta - 12 \cos \theta = 4\) for \(0^\circ \leq \theta \leq 360^\circ\). [4]

(iii) state the largest value of \(k\) for which the equation \(9 \sin \theta - 12 \cos \theta = k\) has any solutions. [1]

Q12.

7 (i) Express \(5 \sin 2\theta + 2 \cos 2\theta\) in the form \(R \sin(2\theta + \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), giving the exact value of \(R\) and the value of \(\alpha\) correct to 2 decimal places. [3]

Hence

(ii) solve the equation
\[5 \sin 2\theta + 2 \cos 2\theta = 4,\]
giving all solutions in the interval \(0^\circ \leq \theta \leq 360^\circ\). [5]

(iii) determine the least value of \(\frac{1}{(10 \sin 2\theta + 4 \cos 2\theta)^2}\) as \(\theta\) varies. [2]

Q13.

8 (i) Prove the identity
\[\frac{1}{\sin(x - 60^\circ) + \cos(x - 30^\circ)} = \csc x.\] [3]

(ii) Hence solve the equation
\[\frac{2}{\sin(x - 60^\circ) + \cos(x - 30^\circ)} = 3 \cot^2 x - 2,\]
for \(0^\circ < x < 360^\circ\). [6]

Q14.
5 The angle \( x \), measured in degrees, satisfies the equation

\[ \cos(x - 30^\circ) = 3 \sin(x - 60^\circ). \]

(i) By expanding each side, show that the equation may be simplified to

\[ (2 \sqrt{3}) \cos x = \sin x. \]

(ii) Find the two possible values of \( x \) lying between \( 0^\circ \) and \( 360^\circ \).

(iii) Find the exact value of \( \cos 2x \), giving your answer as a fraction.

Q15.

4 (i) Express \( \cos \theta + (\sqrt{3}) \sin \theta \) in the form \( R \cos(\theta - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{1}{2} \pi \), giving the exact value of \( \alpha \).

(ii) Hence show that one solution of the equation

\[ \cos \theta + (\sqrt{3}) \sin \theta = \sqrt{2} \]

is \( \theta = \frac{7}{12} \pi \), and find the other solution in the interval \( 0 < \theta < 2 \pi \).

Q16.

3 Find the values of \( x \) satisfying the equation

\[ 3 \sin 2x = \cos x, \]

for \( 0^\circ \leq x \leq 90^\circ \).

Q17.

8 (i) Express \( \cos \theta + \sin \theta \) in the form \( R \cos(\theta - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{1}{2} \pi \), giving the exact values of \( R \) and \( \alpha \).

(ii) Hence show that

\[ \frac{1}{(\cos \theta + \sin \theta)^2} = \frac{1}{2} \sec^2(\theta - \frac{1}{4} \pi). \]

(iii) By differentiating \( \frac{\sin x}{\cos x} \) show that if \( y = \tan x \) then \( \frac{dy}{dx} = \sec^2 x \).

(iv) Using the results of parts (ii) and (iii), show that

\[ \int_0^{\frac{1}{4} \pi} \frac{1}{(\cos \theta + \sin \theta)^2} d\theta = 1. \]
Q18.

3  (i) Express $12 \cos \theta - 5 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 10,$$

giving all solutions in the interval $0^\circ \leq \theta \leq 360^\circ$. [4]

Q19.

4  (i) Prove the identity

$$\tan(x + 45^\circ) - \tan(45^\circ - x) \equiv 2 \tan 2x.$$ [4]

(ii) Hence solve the equation

$$\tan(x + 45^\circ) - \tan(45^\circ - x) = 2,$$

for $0^\circ \leq x \leq 180^\circ$. [3]

Q20.

6  (i) Express $8 \sin \theta - 15 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of $R$ and the value of $\alpha$ correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8 \sin \theta - 15 \cos \theta = 14,$$

giving all solutions in the interval $0^\circ \leq \theta \leq 360^\circ$. [4]

Q21.

4  (i) Show that the equation

$$\sin(x + 30^\circ) = 2 \cos(x + 60^\circ)$$

can be written in the form

$$(3\sqrt{3}) \sin x = \cos x.$$ [3]

(ii) Hence solve the equation

$$\sin(x + 30^\circ) = 2 \cos(x + 60^\circ),$$

for $-180^\circ \leq x \leq 180^\circ$. [3]

Q22.
Q23.

(i) Show that the equation \( \sin(60^\circ - x) = 2 \sin x \) can be written in the form \( \tan x = k \), where \( k \) is a constant.

(ii) Hence solve the equation \( \sin(60^\circ - x) = 2 \sin x \), for \( 0^\circ < x < 360^\circ \).

Q24.

5  Solve the equation \( 8 + \cot \theta = 2 \csc^2 \theta \), giving all solutions in the interval \( 0^\circ \leq \theta \leq 360^\circ \).

Q25.

6  (i) Express \( 3 \cos x + 4 \sin x \) in the form \( R \cos(x - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), stating the exact value of \( R \) and giving the value of \( \alpha \) correct to 2 decimal places.

(ii) Hence solve the equation

\[
3 \cos x + 4 \sin x = 4.5,
\]
giving all solutions in the interval \( 0^\circ < x < 360^\circ \).

Q26.

8  (i) Express \( 2 \sin \theta - \cos \theta \) in the form \( R \sin(\theta - \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), giving the exact value of \( R \) and the value of \( \alpha \) correct to 2 decimal places.

(ii) Hence solve the equation

\[
2 \sin \theta - \cos \theta = -0.4,
\]
giving all solutions in the interval \( 0^\circ \leq \theta \leq 360^\circ \).

(iii) Write down the least value of \( 15 \cos \theta - 9 \sin \theta \) as \( \theta \) varies.

Q27.

5  Solve the equation \( 5 \sec^2 \theta = \tan 2\theta + 9 \), giving all solutions in the interval \( 0^\circ \leq \theta \leq 180^\circ \).

Q28.
3. Solve the equation
\[ 2 \cos 2\theta = 4 \cos \theta - 3, \]
for \(0^\circ \leq \theta \leq 180^\circ\). \[4\]

8. (a) Given that \(\tan A = t\) and \(\tan(A + B) = 4\), find \(\tan B\) in terms of \(t\). \[3\]

(b) Solve the equation
\[ 2 \tan(45^\circ - x) = 3 \tan x, \]
giving all solutions in the interval \(0^\circ \leq x \leq 360^\circ\). \[6\]

7. (i) Express \(3 \cos \theta + \sin \theta\) in the form \(R \cos(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), giving the exact value of \(R\) and the value of \(\alpha\) correct to 2 decimal places. \[3\]

(ii) Hence solve the equation
\[ 3 \cos 2x + \sin 2x = 2, \]
giving all solutions in the interval \(0^\circ \leq x \leq 360^\circ\). \[5\]

3. Solve the equation \(2 \cot^2 \theta - 5 \cosec \theta = 10\), giving all solutions in the interval \(0^\circ \leq \theta \leq 360^\circ\). \[6\]

2. Solve the equation \(3 \sin 2\theta \tan \theta = 2\) for \(0^\circ < \theta < 180^\circ\). \[4\]

Q33.
The angle \( \alpha \) lies between \( 0^\circ \) and \( 90^\circ \) and is such that
\[
2 \tan^2 \alpha + \sec^2 \alpha = 5 - 4 \tan \alpha.
\]

(i) Show that
\[
3 \tan^2 \alpha + 4 \tan \alpha - 4 = 0
\]
and hence find the exact value of \( \tan \alpha \).

(ii) It is given that the angle \( \beta \) is such that \( \cot(\alpha + \beta) = 6 \). Without using a calculator, find the exact value of \( \cot \beta \).

Q34.

7 (i) Express \( 5 \cos \theta - 12 \sin \theta \) in the form \( R \cos(\theta + \alpha) \), where \( R > 0 \) and \( 0^\circ < \alpha < 90^\circ \), giving the value of \( \alpha \) correct to 2 decimal places.

(ii) Hence solve the equation \( 5 \cos \theta - 12 \sin \theta = 8 \) for \( 0^\circ < \theta < 360^\circ \).

(iii) Find the greatest possible value of
\[
7 + 5 \cos \frac{1}{2} \phi - 12 \sin \frac{1}{2} \phi
\]
as \( \phi \) varies, and determine the smallest positive value of \( \phi \) for which this greatest value occurs.

P3 (variant1 and 3)

Q1.

2 Solve the equation
\[
\sin \theta = 2 \cos 2\theta + 1,
\]
giving all solutions in the interval \( 0^\circ \leq \theta \leq 360^\circ \).

Q2.

3 Solve the equation
\[
\tan(45^\circ - x) = 2 \tan x,
\]
giving all solutions in the interval \( 0^\circ < x < 180^\circ \).

Q3.
Q4.

(i) Prove the identity \( \cos 4\theta + 4 \cos 2\theta = 8 \cos^4 \theta - 3 \). [4]

(ii) Hence

(a) solve the equation \( \cos 4\theta + 4 \cos 2\theta = 1 \) for \(-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi\), [3]

(b) find the exact value of \( \int_0^{\frac{1}{2}\pi} \cos^4 \theta \, d\theta \). [3]

Q5.

(i) Show that the equation

\[
\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k
\]

can be written in the form

\[
(2\sqrt{3})(1 + \tan^2 \theta) = k(1 - 3\tan^2 \theta).
\] [4]

(ii) Hence solve the equation

\[
\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},
\]
giving all solutions in the interval \(0^\circ \leq \theta \leq 180^\circ\). [3]

Q6.

(i) Express \( 4 \cos \theta + 3 \sin \theta \) in the form \( R \cos(\theta - \alpha) \), where \( R > 0 \) and \( 0 < \alpha < \frac{1}{2}\pi \). Give the value of \( \alpha \) correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation \( 4 \cos \theta + 3 \sin \theta = 2 \) for \( 0 < \theta < 2\pi \), [4]

(b) find \( \int \frac{50}{(4 \cos \theta + 3 \sin \theta)^2} \, d\theta \). [3]

Q7.
Q8.

3. Solve the equation \( \cos(\theta + 60^\circ) = 2 \sin \theta \), giving all solutions in the interval \( 0^\circ \leq \theta \leq 360^\circ \). [5]

Q9.

8. (i) Express \((\sqrt{6}) \cos \theta + (\sqrt{10}) \sin \theta\) in the form \(R \cos(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). Give the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle \(\theta\) which satisfies the equation

(a) \((\sqrt{6}) \cos \theta + (\sqrt{10}) \sin \theta = -4\). [2]

(b) \((\sqrt{6}) \cos \frac{\theta}{2} + (\sqrt{10}) \sin \frac{\theta}{2} = 3\). [4]

Q10.

6. (i) Express \(\cos x + 3 \sin x\) in the form \(R \cos(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\), giving the exact value of \(R\) and the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence solve the equation \(\cos 2\theta + 3 \sin 2\theta = 2\), for \(0^\circ < \theta < 90^\circ\). [5]

Q11.

3. (i) Express \(8 \cos \theta + 15 \sin \theta\) in the form \(R \cos(\theta - \alpha)\), where \(R > 0\) and \(0^\circ < \alpha < 90^\circ\). Give the value of \(\alpha\) correct to 2 decimal places. [3]

(ii) Hence solve the equation \(8 \cos \theta + 15 \sin \theta = 12\), giving all solutions in the interval \(0^\circ < \theta < 360^\circ\). [4]

Q12.

3. Solve the equation \(\sin(\theta + 45^\circ) = 2 \cos(\theta - 30^\circ)\), giving all solutions in the interval \(0^\circ < \theta < 180^\circ\). [5]

Q13.
Q14.

2 (i) Express $24 \sin \theta - 7 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of $\alpha$ correct to 2 decimal places. [3]

(ii) Hence find the smallest positive value of $\theta$ satisfying the equation $24 \sin \theta - 7 \cos \theta = 17$. [2]

Q15.

7 (i) Given that $\sec \theta + 2 \cosec \theta = 3 \cosec 2\theta$, show that $2 \sin \theta + 4 \cos \theta = 3$. [3]

(ii) Express $2 \sin \theta + 4 \cos \theta$ in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the value of $\alpha$ correct to 2 decimal places. [3]

(iii) Hence solve the equation $\sec \theta + 2 \cosec \theta = 3 \cosec 2\theta$ for $0^\circ < \theta < 360^\circ$. [4]

Q16.

3 (i) Show that the equation $\tan(x - 60^\circ) + \cot x = \sqrt{3}$ can be written in the form $2 \tan^2 x + (\sqrt{3}) \tan x - 1 = 0$. [3]

(ii) Hence solve the equation $\tan(x - 60^\circ) + \cot x = \sqrt{3}$, for $0^\circ < x < 180^\circ$. [3]

Q17.
8

(i) By first expanding \( \sin(2\theta + \theta) \), show that
\[
\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta.
\] [4]

(ii) Show that, after making the substitution \( x = \frac{2 \sin \theta}{\sqrt{3}} \), the equation \( x^3 - x + \frac{1}{6} \sqrt{3} = 0 \) can be written in the form \( \sin 3\theta = \frac{1}{4} \). [1]

(iii) Hence solve the equation
\[
x^3 - x + \frac{1}{6} \sqrt{3} = 0,
\]
giving your answers correct to 3 significant figures. [4]

Q18.

4

(i) Show that \( \cos(\theta - 60^\circ) + \cos(\theta + 60^\circ) \equiv \cos \theta \). [3]

(ii) Given that \( \frac{\cos(2x - 60^\circ) + \cos(2x + 60^\circ)}{\cos(x - 60^\circ) + \cos(x + 60^\circ)} = 3 \), find the exact value of \( \cos x \). [4]